AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

BY

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CHAPTER I

NATURE AND DESIGN OF THIS WORK

1. The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; to make that method itself the basis of a general method for the application of the mathematical doctrine of Probabilities; and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and constitution of the human mind.

2. That this design is not altogether a novel one it is almost needless to remark, and it is well known that to its two main practical divisions of Logic and Probabilities a very considerable share of the attention of philosophers has been directed. In its ancient and scholastic form, indeed, the subject of Logic stands almost exclusively associated with the great name of Aristotle. As it was presented to ancient Greece in the partly technical, partly metaphysical disquisitions of the Organon, such, with scarcely any essential change, it has continued to the present day. The stream of original inquiry has rather been directed towards questions of general philosophy, which, though they [2] have arisen

conceivable forms of language; and adopting this principle, let the following classification be considered.

CLASS I

5. *Appellative or descriptive signs, expressing either the name of a thing, or some quality or circumstance belonging to it.*

To this class we may obviously refer the substantive proper or common, and the adjective. These may indeed be regarded as differing only in this respect, that the former expresses the substantive existence of the individual thing or things to which it refers; the latter implies that existence. If we attach to the adjective the universally understood subject "being" or "thing," it becomes virtually a substantive, and may for all the essential purposes of reasoning be replaced by the substantive. Whether or not, in every particular of the mental regard, it is the same thing to say, "Water is a fluid thing," as to say, "Water is fluid"; it is at least equivalent in the expression of the processes of reasoning.

[28] It is clear also, that to the above class we must refer any sign which may conventionally be used to express some circumstances or relation, the detailed exposition of which would involve the use of many signs. The epithets of poetic diction are very frequently of this kind. They are usually compounded adjectives, singly fulfilling the office of a many-worded description. Homer's "deep-eddying ocean" embodies a virtual description in the single word βαθυδίνης. And conventionally any other description addressed either to the imagination or to the intellect might equally be represented by a single sign, the use of which would in all essential points be subject to the same laws as the use of the adjective "good" or "great." Combined with the subject "thing," such a sign would virtually become a substantive; and by a single substantive the combined meaning both of thing and quality might be expressed.

6. Now, as it has been defined that a sign is an arbitrary mark, it is permissible to replace all signs of the species above described by letters. Let us then agree to represent the class of individuals to which a particular name or description is applicable, by a single letter, as x. If the name is "men," for instance, let x represent "all men," or the class "men." By a class is usually meant a collection of individuals, to each of which a particular name or description may be applied; but in this work the meaning of the term will be extended so as to include the case in which but a single individual exists, answering to the required name or description, as well as the cases denoted by the terms "nothing" and "universe," which as "classes" should be understood to comprise respectively "no beings," "all beings." Again, if an adjective, as "good," is employed as a term of description, let us represent by a letter, as y, all things to which the description "good" is applicable, *i.e.* "all good things," or the class "good things." Let it further be agreed, that by the combination xy shall be represented that class of things to which the names or descriptions represented by x and y are simultaneously applicable. Thus, if x alone stands for "white things," and y for "sheep," let xy stand for "white sheep"; and in like manner, if z stand for "horned things," and x and y retain their previous interpretations, let zxy represent [29] "horned white sheep," *i.e.* that collection of things to which the name "sheep," and the descriptions "white" and "horned" are together applicable.

Let us now consider the laws to which the symbols x, y, &c., used in the above sense, are subject.

7. First, it is evident, that according to the above combinations, the order in which two symbols are written is indifferent. The expressions xy and yx equally represent that class of things to the several members of which the names or descriptions x and y are together applicable. Hence we have,

$$xy = yx. \quad (1)$$

minerals," "barren mountains, or fertile vales," are examples of this kind. In strictness, the words [33] "and," "or," interposed between the terms descriptive of two or more classes of objects, imply that those classes are quite distinct, so that no member of one is found in another. In this and in all other respects the words "and" "or" are analogous with the sign + in algebra, and their laws are identical. Thus the expression "men and women" is, conventional meanings set aside, equivalent with the expression "women and men." Let x represent "men," y, "women"; and let + stand for "*and*" and "*or*," then we have

$$x+y=y+x, \qquad (3)$$

an equation which would equally hold true if x and y represented *numbers*, and + were the sign of arithmetical addition.

Let the symbol z stand for the adjective "European," then since it is, in effect, the same thing to say "European men and women," as to say "European men and European women," we have

$$z(x+y)=zx+zy. \qquad (4)$$

And this equation also would be equally true were x, y, and z symbols of number, and were the juxtaposition of two literal symbols to represent their algebraic product, just as in the logical signification previously given, it represents the class of objects to which both the epithets conjoined belong.

The above are the laws which govern the use of the sign +, here used to denote the positive operation of aggregating parts into a whole. But the very idea of an operation effecting some positive change seems to suggest to us the idea of an opposite or negative operation, having the effect of undoing what the former one has done. Thus we cannot conceive it possible to collect parts into a whole, and not conceive it also possible to separate a part from a whole. This operation we express in common language by the sign *except*, as, "All men *except* Asiatics," "All states *except* those which are monarchical." Here it is implied that the

things excepted form a part of the things from which they are excepted. As we have expressed the operation of aggregation by the sign +, so we may express the negative operation above described by − minus. Thus if x be taken to represent men, and y, Asiatics, *i.e.* Asiatic men, [34] then the conception of "All men except Asiatics" will be expressed by $x-y$. And if we represent by x, "states," and by y the descriptive property "having a monarchical form," then the conception of "All states except those which are monarchical" will be expressed by $x-xy$.

As it is indifferent for all the *essential* purposes of reasoning whether we express excepted cases first or last in the order of speech, it is also indifferent in what order we write any series of terms, some of which are affected by the sign −. Thus we have, as in the common algebra,

$$x-y=-y+x. \qquad (5)$$

Still representing by x the class "men," and by y "Asiatics," let z represent the adjective "white." Now to apply the adjective "white" to the collection of men expressed by the phrase "Men except Asiatics," is the same as to say, "White men, except white Asiatics." Hence we have

$$z(x-y)=zx-zy. \qquad (6)$$

This is also in accordance with the laws of ordinary algebra.

The equations (4) and (6) may be considered as exemplification of a single general law, which may be stated by saying, *that the literal symbols, x, y, z, &c. are distributive in their operation.* The general fact which that law expresses is this, viz.:—If any quality or circumstance is ascribed to all the members of a group, formed either by aggregation or exclusion of partial groups, the resulting conception is the same as if the quality or circumstance were first ascribed to each member of the partial groups, and the aggregation or exclusion effected afterwards. That which is ascribed to the members of the whole is ascribed to the members of all its parts, howsoever those parts are connected together.