# Bayesian network handout

Note Title

Example calculations for the alarm network, Fig 14.2 p512.
(All calculations to 3 significant figures.)

(1) $P(B) = 0.001$ $P(\neg B) = 0.999$

(2) $P(E) = 0.002$ $P(\neg E) = 0.998$

(3) $P(A \mid B, E) = 0.95$ $P(\neg A \mid B, E) = 0.05$
$P(A \mid B, \neg E) = 0.94$ $P(\neg A \mid B, \neg E) = 0.06$
$P(A \mid \neg B, E) = 0.29$ $P(\neg A \mid \neg B, E) = 0.71$
$P(A \mid \neg B, \neg E) = 0.001$ $P(\neg A \mid \neg B, \neg E) = 0.999$

read directly off the figure

(4) $P(A, E \mid B) = \underbrace{P(A \mid E, B)}_{=0.95 \text{ from above}} \underbrace{P(E \mid B)}$ → follows from definition of conditional probability, with a "$\mid B$" added everywhere: $P(X,Y) = P(X \mid Y)\,P(Y)$, so $P(X, Y \mid B) = P(X \mid Y, B)\,P(Y \mid B)$

look at the network – E and B are independent – so $P(E \mid B) = P(E) = 0.002$

$= 0.95 \times 0.002$
$= 0.0019$

$P(A, \neg E \mid B) = P(A \mid \neg E, B)\, P(\neg E \mid B)$ (same reasoning as above)
$= 0.94 \times 0.998$
$= 0.938$

(5) $P(A|B) = P(A, E|B) + P(A, \neg E|B)$ ] ← sum over all possibilities of other relevant variables

$= 0.0019 + 0.938$ ] ← from (4)

$= 0.940$

$P(\neg A|B) = 1 - P(A|B)$ ] ← by definition

$= 0.060$

(6) $P(J|B) = P(J, A|B) + P(J, \neg A|B)$ ] ← sum over all possibilities of other relevant variables

$= P(J|A,B)P(A|B) + P(J|\neg A,B)P(\neg A|B)$ ] (from definition of cond prob, as in (4))

using conditional independence in the network

$= P(J|A)\,P(A|B) + P(J|\neg A)\,P(\neg A|B)$

$= 0.9 \times 0.940 + 0.05 \times 0.060$

$= 0.849$

$P(\neg J|B) = 1 - P(J|B) = 1 - 0.849$

$= 0.151$

⑦ $P(J \mid \neg B)$ = [all reasoning is analogous to ⑥ and is omitted]

$$= P(J, A \mid \neg B) + P(J, \neg A \mid \neg B)$$

$$= P(J \mid A)\, P(A \mid \neg B) + P(J \mid \neg A)\, P(\neg A \mid \neg B)$$

$$P(A \mid \neg B) = P(A, E \mid \neg B) + P(A, \neg E \mid \neg B)$$

$$= P(A \mid E, \neg B) P(E) + P(A \mid \neg E, \neg B) P(\neg E)$$

$$= 0.29 \times 0.002 + 0.001 \times 0.998$$

$$= 0.00158$$

$$P(\neg A \mid \neg B) = 1 - P(A \mid \neg B) = 1 - 0.00158$$

$$= 0.998$$

$$= 0.9 \times 0.00158 + 0.05 \times 0.998$$

$$= 0.0513$$

$$P(\neg J \mid \neg B) = 1 - P(J \mid \neg B) = 1 - 0.0513$$

$$= 0.949$$

(8) $P(A|\neg B) = P(A, E|\neg B) + P(A, \neg E|\neg B)$

$= P(A|E, \neg B)\, P(E) + P(A|\neg E, \neg B)\, P(\neg E)$

$= 0.29 \times 0.002 + 0.001 \times 0.998$

$= 0.00158$

same reasoning as (4) and (5) combined

$P(\neg A|\neg B) = 1 - P(A|\neg B)$

$= 0.998$

(9) $P(B|J) = \underbrace{P(J|B)}_{\text{know from (6)}}\ \underbrace{P(B)}_{\text{know from (1)}} / P(J)$ (Bayes' rule)

Could calculate P(J) explicitly, but this is a lengthy calculation. We will calculate it implicitly, later. Leave it as P(J) for now.

$= 0.849 \times 0.001 / P(J)$

$= 0.000849 / P(J)$

$P(\neg B|J) = \underbrace{P(J|\neg B)}_{\text{know from (7)}}\ \underbrace{P(\neg B)}_{\text{know from (1)}} / \underbrace{P(J)}_{\text{leave as P(J) for now}}$ (Bayes' rule)

$= 0.0513 \times 0.999 / P(J)$

$= 0.0512 / P(J)$

Now, we know that

$$P(B|J) + P(\neg B|J) = 1$$

Thus, we have:

$$0.000849 / P(J) + 0.0512 / P(J) = 1$$

Rearrange: $P(J) = 0.000849 + 0.0512$
$= 0.0521$ ← thus, $P(J)$ has been calculated !!

Finally, we have

$$P(B|J) = 0.000849 / P(J) = 0.000849 / 0.0521$$
$$= 0.0163$$

Sample quiz exercise: Compute 8 and 9, given the results from 1 to 7.