Boolean Algebra and Logic Gates

Note Title

Part 1: Boolean Algebra

Boolean algebra originates from George Boole, 19th Century British mathematician and philosopher — see excerpt from his Laws of Thought on resources page.

Boolean algebra deals with 2 states: true, false
on, off
1, 0

Boolean variable: variable that can take one of two values (typically 0 or 1).

Boolean function: - inputs are one or more Boolean variables.
- output is a single Boolean variable.

[why are we interested? Because this is all computers do, all day every day!!]

Can define a Boolean function by a truth table

e.g. $f(a,b)$ defined by

a	b	f(a,b)
0	0	0
0	1	1
1	0	0
1	1	0

e.g. $g(a,b,c)$ defined by

a	b	c	$g(a,b,c)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Elementary Boolean functions include:

AND

a	b	a AND b
0	0	0
0	1	0
1	0	0
1	1	1

OR

a	b	a OR b
0	0	0
0	1	1
1	0	1
1	1	1

NOT

a	NOT a
0	1
1	0

Notation varies quite a bit:

a AND b $\equiv$ ab $\equiv a \cdot b \equiv a \wedge b \equiv$ a && b

a OR b $\equiv a + b \equiv a \vee b \equiv$ a || b

NOT a $\equiv \bar{a} \equiv a' \equiv \neg a \equiv \neg a \equiv$!a

architecture text book

AI text book

Java

some logic books even use a+b for a AND b !

BEWARE!!

We can combine elementary Boolean functions to get any Boolean function

e.g. $\bar{a}b$:

a	b	$\bar{a}$	$\bar{a}b$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

looking back, we see $f(a,b) = \bar{a}b$.

[Demo TruthTable.java]

Activity: Consider the Boolean function $c\bar{a}\bar{b} + a\bar{b}\bar{c}$.

1. work out the truth table by hand
2. check using TruthTable.java
3. look back – is this function the same as one we've seen already?

optional:

4. try $\bar{b}(c\bar{a} + a\bar{c})$

   is this the same as something we've seen?

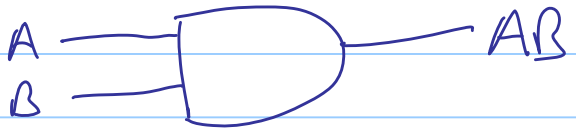
→ important point: same function can be represented by different formulas

e.g. $\bar{b}(c\bar{a} + a\bar{c}) = ca\bar{b} + a\bar{b}\bar{c}$

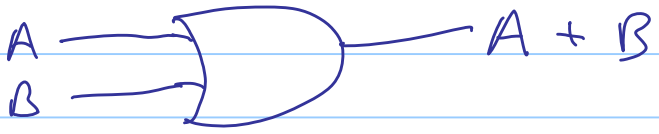
Part 2: Logic Gates

- Computer circuits are built out of gates that implement simple Boolean functions.
- A gate is built from a few transistors.
- Common gates and their circuit symbols are:

AND



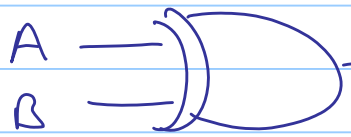
OR



NOT

↑ sometimes this "O" represents a NOT attached to another gate

XOR

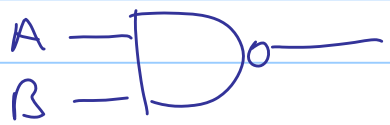


$A\bar{B} + \bar{A}B$ i.e. $\equiv A \oplus B$

↳ Symbol for XOR

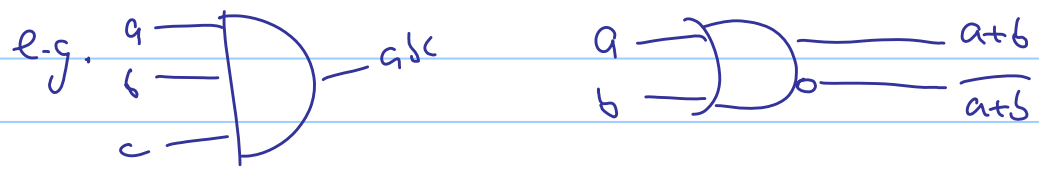
A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

NAND



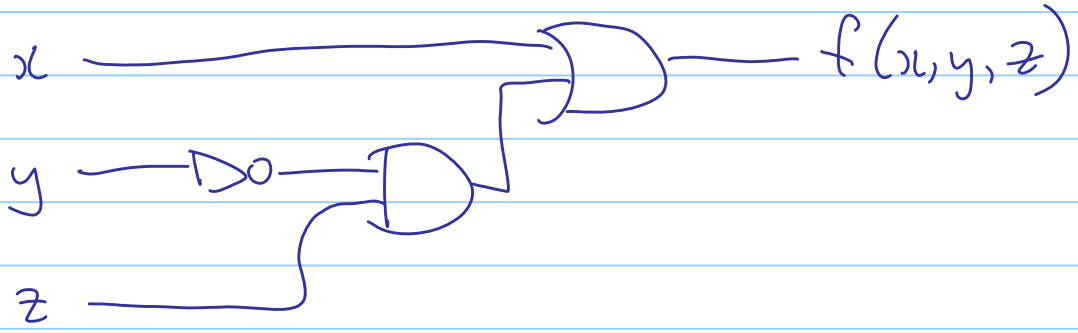
NOR

- NAND is a universal gate ie. you can build all others from it.
- NOR is also universal.
- gates can have multiple inputs or outputs



- Any Boolean function can be implemented using a combination of gates

e.g. $f(x, y, z) = x + \bar{y}z$



ACTIVITY: build f using SimCir.