Combinational circuits

Note Title

Last time:

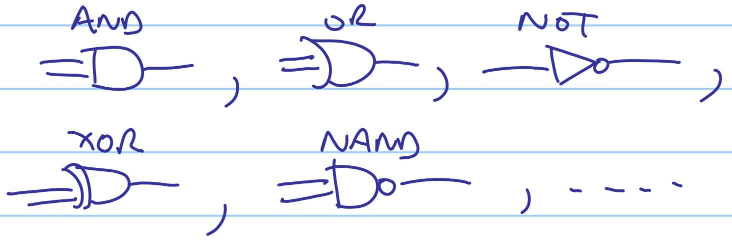
- Boolean functions e.g. $f(a,b,c) = ab + a\bar{c} + \overline{bc}$

truth table:

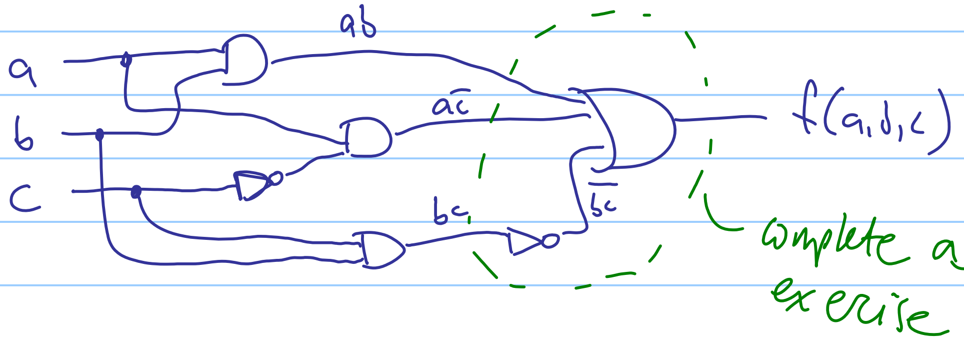
a	b	c	ab	$a\bar{c}$	$\overline{bc}$	f
0	0	0	0	0	1	1
0	0	1	0	0	1	1
0	1	0	0	0	1	1
0	1	1	0	0	0	0
1	0	0	0	1	1	1
1	0	1	0	0	1	1
1	1	0	1	1	1	1
1	1	1	1	0	0	1

fill in as exercise

- Logic gates e.g.



- Boolean functions as circuits



complete as an exercise

Demo: TruthTable.java
with $a + \bar{b} + \bar{c}$

Today: Combinational circuits

- they produce outputs (almost instantaneously) from present inputs (do not depend on past inputs)

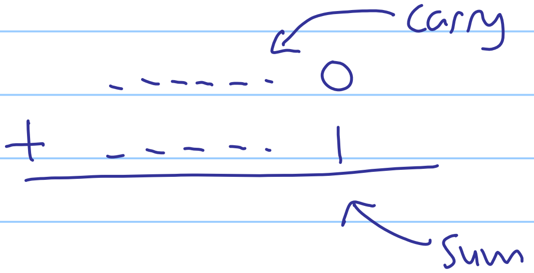
- any boolean function can be implemented as a combinatoral circuit

- every combinational circuit computes a boolean function.

- we study specific, useful circuits:
  ① half adder
  ② full adder
  ③ decoder
  ④ multiplexer

① Half adder

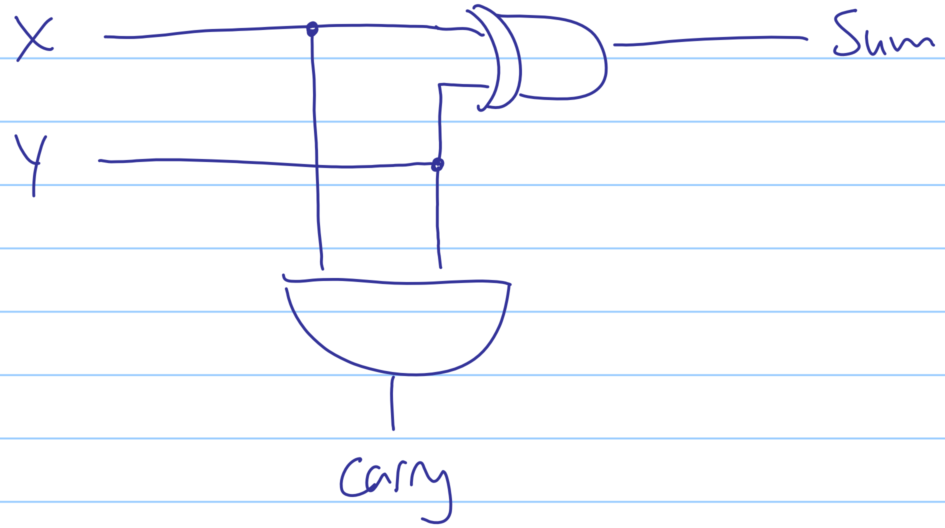
motivation: computes last digit of binary add



Inputs		Outputs	
X	Y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

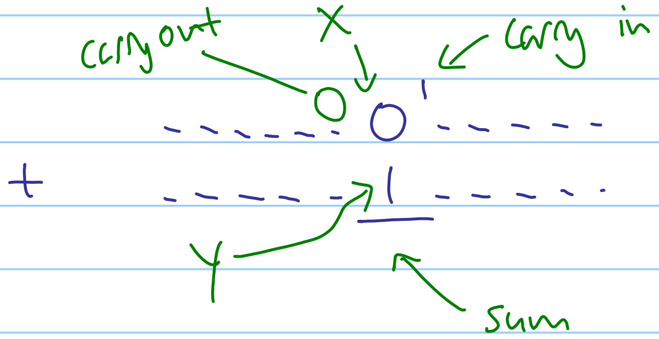
single gate?
XOR

single gate?
AND

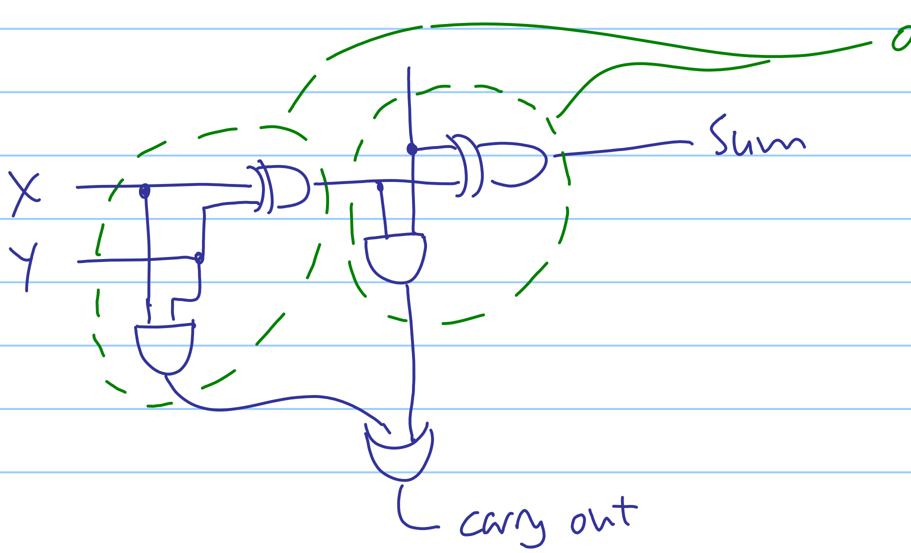


② Full adder

motivation : middle digit of binary add



Inputs			Outputs	
X	Y	Carry In	Sum	Carry out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

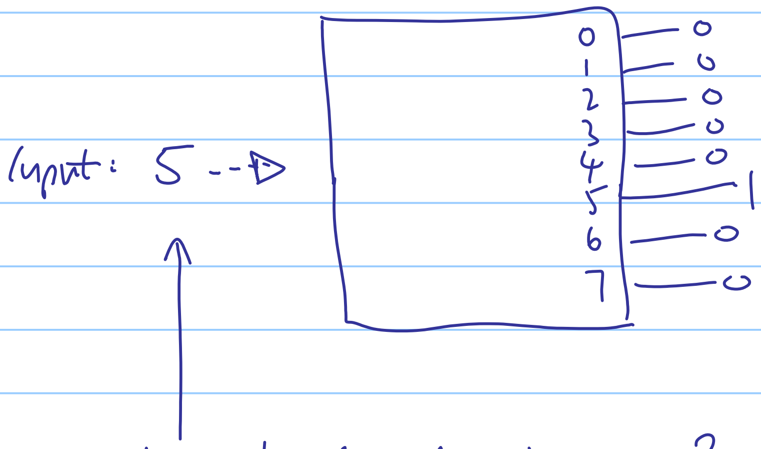


Note: Ripple-carry adder.
Slide 3.35
or fig 3.13

③ Decoder

- Input describes a number in some range, eg. $n \in \{0, 1, 2, \dots 63\}$
- Output has one line for each possible input
- When input is n, the nth output line is on, all others are off

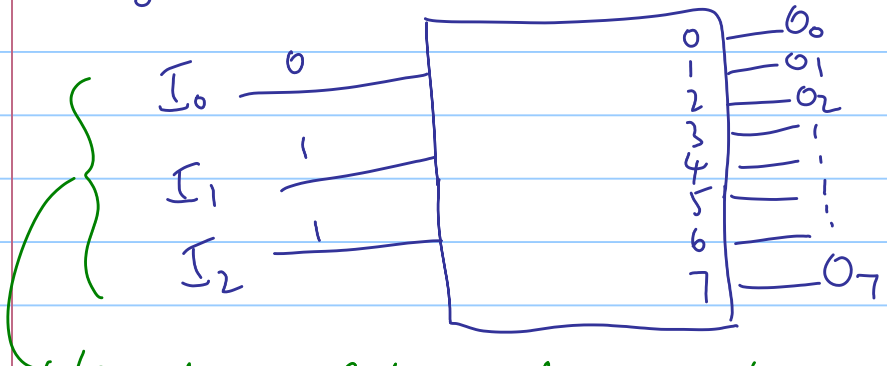
Motivation: Useful for activating a memory location based on its numerical address



how to describe this input?

Represent n as unsigned int. Need 3 input lines

e.g. $n = 6$

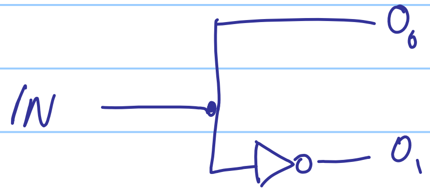


note ordering of bits and interpretation. See fig 3.15 in textbook

Activity: Solve decoder puzzle on resources page.

ACTIVITY: create a 1-to-2 decoder

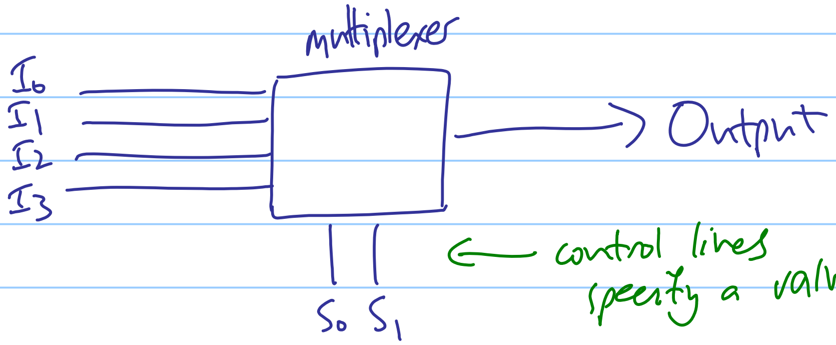
Answer:



④ Multiplexer

- control lines specify a number in some range
  e.g. $n \in \{0,1,..31\}$
- input has one line for each control value.
- the single output is a copy of the nth input

useful for e.g. selecting a desired input on a data projector.



← control lines specify a value in $\{0,1,2,3\}$

Activity: solve the multiplexer puzzle on resources page

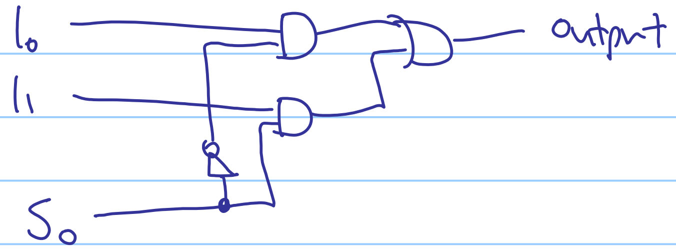
See also: bit shifter circuit (consult textbook & puzzles handout)

Objective is to shift the input left or right

e.g. 1101 shifted left is 1010 (fill in with 0)
shifted right is 0110 (fill in with 0)

Activity: create a 2-to-1 multiplexer in SimCir.

Solution:



Challenge: Build a ripple carry adder for 4-bit addition.

Also, see online demos on resources page.