First-order Logic

Note Title

Motivation

PL (propositional logic) is inefficient at representing certain commonly-occurring concepts.

e.g. (a) Describing the fact that all the students in this class are CS majors

we need atoms: P_1: Chris is a CS major

$\vdots$

P_9: Danielle is a CS major

KB is: $P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_9$

Easier is: $\forall x \; CSMajor(x)$

"for all"

(b) Describe "Someone in the class is an Astronomy minor"

we need atoms: Q_1: Chris is an Astronomy minor

$\vdots$

Q_9: Danielle is an Astronomy minor

KB is $Q_1 \vee Q_2 \vee \ldots \vee Q_9$.

Easier is: $\exists x \; AstronomyMinor(x)$

there exists

Thus, the basic idea behind <u>First-Order Logic</u> (FOL) is to add <u>quantifiers</u> ($\forall, \exists$) to propositional logic.

In more detail, FOL uses:

- <u>objects</u> – the elements in the domain of discourse (e.g. students in this class)

- <u>relations</u> – basic statements about the objects that can be true or false

  e.g. CSMajor(Omar)
  Helped(Justine, Sam)
  Team(Cole, Cooper, Danielle)

- <u>functions</u> – input is one or more objects, <u>output is an object</u>

  e.g. $\underbrace{\text{ProgAsstPartner(Omar)}}_{\text{evaluates to Justine}}$

- <u>quantifiers</u> – $\forall, \exists$.

More details on relations:

- a unary relation is a property

  e.g. MathMajor(Nick) means "Nick is a math major"
  Blue(Sky) means "The sky is blue"

- binary relations follow an important ordering convention:
  P(X, Y) often means "X is a P of Y"
  or "X P Y"

  e.g. Instructor(John, Cooper) means
  "John is the instructor of Cooper"
  Helped(Chris, Sam) means
  "Chris helped Sam"

BIG WARNING: Functions and relations are completely different, but look the same.

Functions return objects
Relations return true or false

Examples:

1. Father(John) means:

relation	function

fill in as exercise

Son (Leslie, Rowan) means

fill in as exercise

relation	function

De Morgan's rules for quantifiers

- $\neg \forall x \; P(x) \equiv \exists x \; \neg P(x)$

  e.g. not all students in this class are CS majors

  $\equiv$

  fill in as exercise

- $\neg \exists x \; P(x) \equiv \forall x \; \neg P(x)$

  e.g. no student in the class is an Astronomy Minor

  $\equiv$

  fill in as exercise

Note: follows easily from PL version of De Morgan, since $\forall$ is like a conjunction and $\exists$ is like a disjunction.

Meaning of equals: The '=' sign means "the same object"

e.g. ProgAsstPartner (Justine) = Omar

Important examples from text book: 8.10, 8.11, 8.24

For interest only: see real-world applications of FOL on resources web page.