

# First-order Logic

Note Title

## Motivation

PL (propositional logic) is inefficient at representing certain commonly-occurring concepts.

e.g. (a) Describing the fact that all the students in this class are CS majors

we need atoms:  $P_1$ : Chris is a CS major

⋮

$P_q$ : Danielle is a CS major

KB is:  $P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_q$

Easier is:  $\forall x \text{ CSMajor}(x)$

↖ "for all"

(b) Describe "Someone in the class is an Astronomy minor"

we need atoms:  $Q_1$ : Chris is an Astronomy minor

⋮

$Q_q$ : Danielle is an Astronomy minor

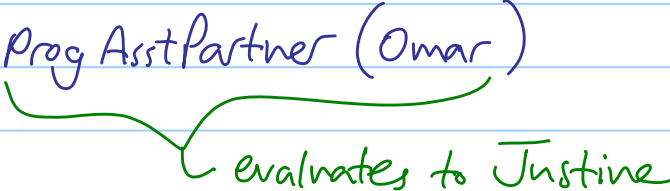
KB is  $Q_1 \vee Q_2 \vee \dots \vee Q_q$

Easier is:  $\exists x \text{ AstronomyMinor}(x)$

↖ there exists

Thus, the basic idea behind First-order Logic (FOL) is to add quantifiers ( $\forall, \exists$ ) to propositional logic.

In more detail, FOL uses:

- objects - the elements in the domain of discourse  
(e.g. students in this class)
- relations - basic statements about the objects that can be true or false  
e.g. CSMajor (Omar)  
Helped (Justine, Sam)  
Team (Cole, Cooper, Danielle)
- functions - input is one or more objects, output is an object  
e.g. ProgAsstPartner (Omar)  
evaluates to Justine
- quantifiers -  $\forall, \exists$ .

More details on relations:

- a unary relation is a property

e.g. MathMajor(Nick) means "Nick is a math major"  
Blue(Sky) means "The sky is blue"

- binary relations follow an important ordering convention:  
 $P(X, Y)$  often means "X is a P of Y"  
or "X P Y"

e.g. Instructor(John, Cooper) means  
"John is the instructor of Cooper"  
Helped(Chris, Sam) means  
"Chris helped Sam"

BIG WARNING: Functions and relations are completely different, but look the same.

Functions return objects  
Relations return true or false

Examples:

1. Father(John) means:

relation	function

fill in as exercise. →

Son (Leslie, Rowan) means

fill in as  
exercise →

relation	function

## De Morgan's rules for quantifiers

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

e.g. not all students in this class are CS majors

≡

fill in as exercise

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

e.g. no student in the class is an Astronomy Minor

≡

fill in as exercise

Note: follows easily from PL version of De Morgan, since  $\forall$  is like a conjunction and  $\exists$  is like a disjunction.

Meaning of equals: The (=) sign means "the same object"  
e.g. ProgAristPartner (Justine) = Omar

Important examples from text book: 8.10, 8.11, 8.24

for interest only: see real-world applications of FOL  
on resources web page.