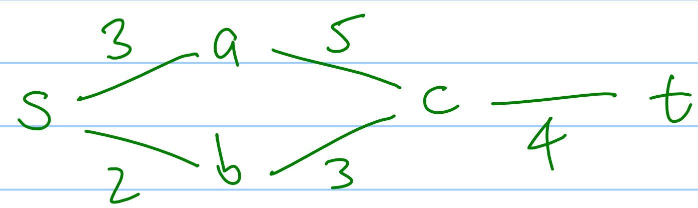
Lecture notes for AI class 5, "Heuristics for informed search"

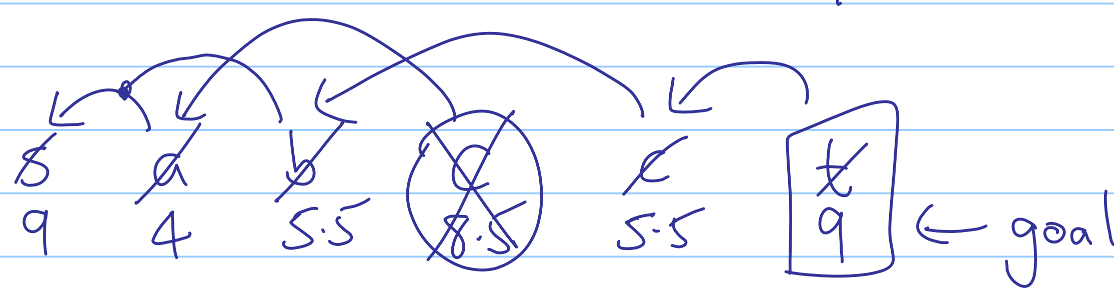
Note Title

First, some simple examples to get us thinking.
Want to get from source s to target t on following map:



Example 1 Use heuristic:

n	h(n)
s	9
a	1
b	3.5
c	0.5

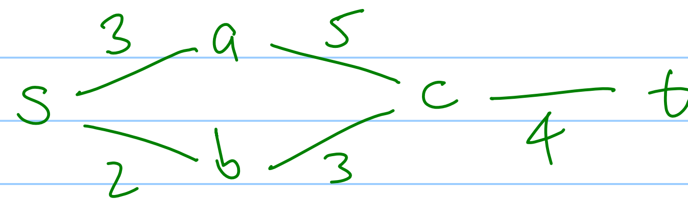


Note the need to <u>replace</u> c in the frontier.

(as described in text book – see last line of fig 3.14)

Example 2

Same map:



Different heuristic:

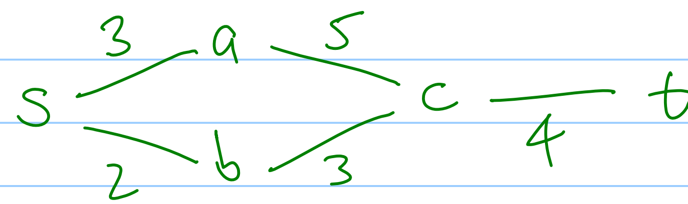
n	h(n)
S	9
a	4
b	7
c	0.5

~~S~~	~~a~~	b	~~c~~	t
9	7	9	8.5	12

c is already explored, so nothing to expand. End up choosing nonoptimal S–a–c–t path!

Note: this would have worked if we used <u>tree</u> search rather than <u>graph</u> search.

Example 3   Same map:



Different heuristic:

n	h(n)
s	9
a	4
b	11
c	0.5

~~s~~	~~a~~	b	~~c~~	$\boxed{\cancel{t}\ 12}$	
9	7	13	8.5	12	← goal

Chose non-optimal s–a–c–t path!
Problem persists even with tree search!

What went wrong?
We got stuck at b – b wasn't expanded because its cost was overestimated.

FACT: If h never overestimates, A* tree search is optimal

this is called "admissible"

Definition If for all states s,
$h(s) \leq$ optimal cost to goal from s
then h is admissible

e.g. straight-line distances on a map are admissible.

Fact A^* tree search is optimal for admissible heuristics.

Question: which of the earlier heuristics were admissible?

Definition If, for all pairs of states s, s' where s' is generated by applying some action to s, we have

$$h(s) \leq (\text{cost from } s \text{ to } s') + h(s')$$

then h is consistent.

e.g. straight-line distances on a map are consistent because of triangle inequality.

Fact A^* graph search is optimal for consistent heuristics.

Fact Consistent $\Rightarrow$ admissible

proof: Suppose h is consistent and let $s_0, s_1, s_2, \dots s_n$ be an optimal path from s_0 to s_n.

Then

$$h(s_0) \le cost(s_0, s_1) + h(s_1)$$
$$\le cost(s_0, s_1) + cost(s_1, s_2) + h(s_2)$$
$$\vdots$$
$$\le \sum_{i=0}^{n-1} cost(s_i, s_{i+1})$$
$$= \text{optimal cost to goal.}$$

by defn of consistency

QED. $\square$