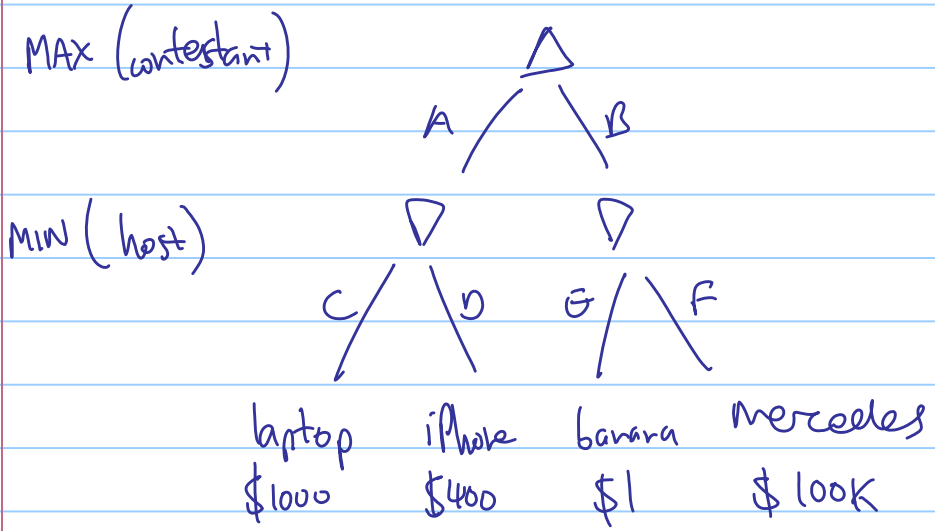
α-β Pruning Lecture notes

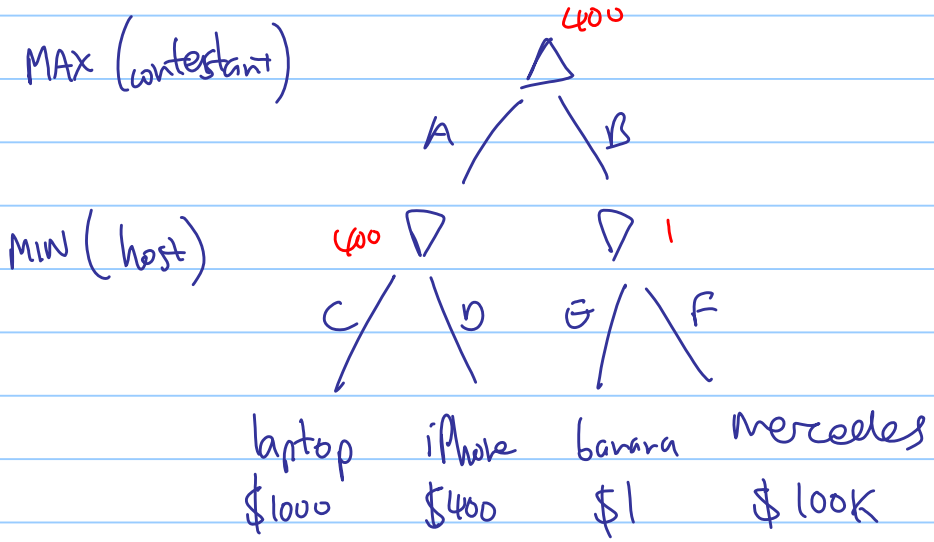
Note Title

(1) Revision of minimax

gameshow example:



(a) informal version: just 'bubble up' best choices from the bottom

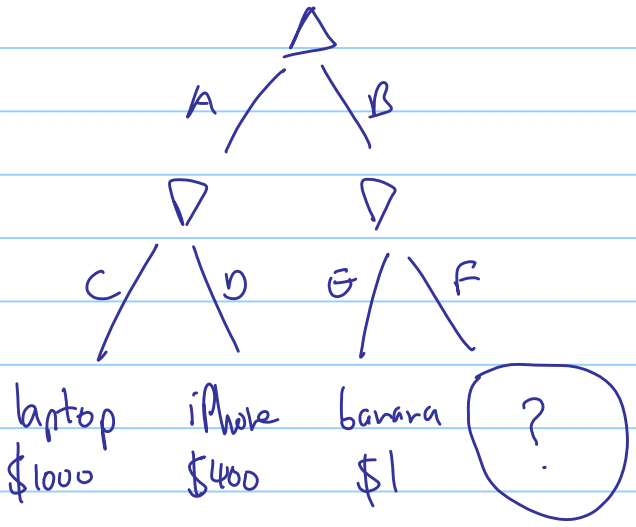


now do more realistic game on handout

(b) formal version: - see textbook fig 5.3
- note the mutually recursive functions

(2) α-β pruning

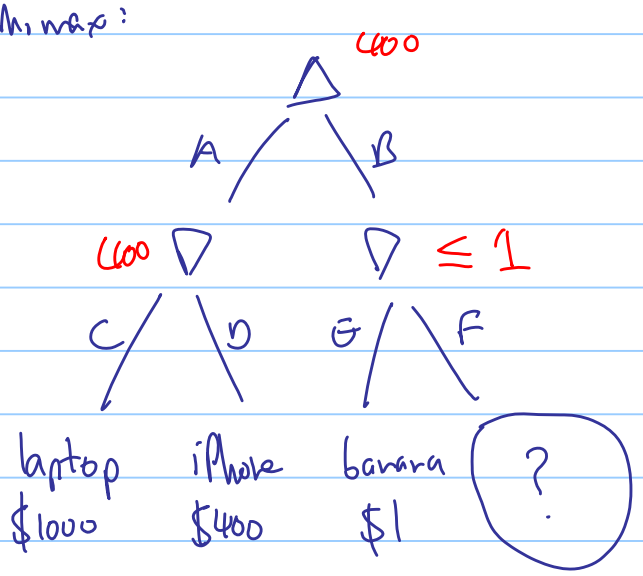
Imagine walking the set beforehand, but the host refuses to open door F, so we have:



Does this change your strategy?
think: what if F contains
- another banana?
- a nickel?
- a mercedes?

Interesting fact: Contents of F does not affect your strategy (against a rational opponent), because the iPhone is better than the banana. !!

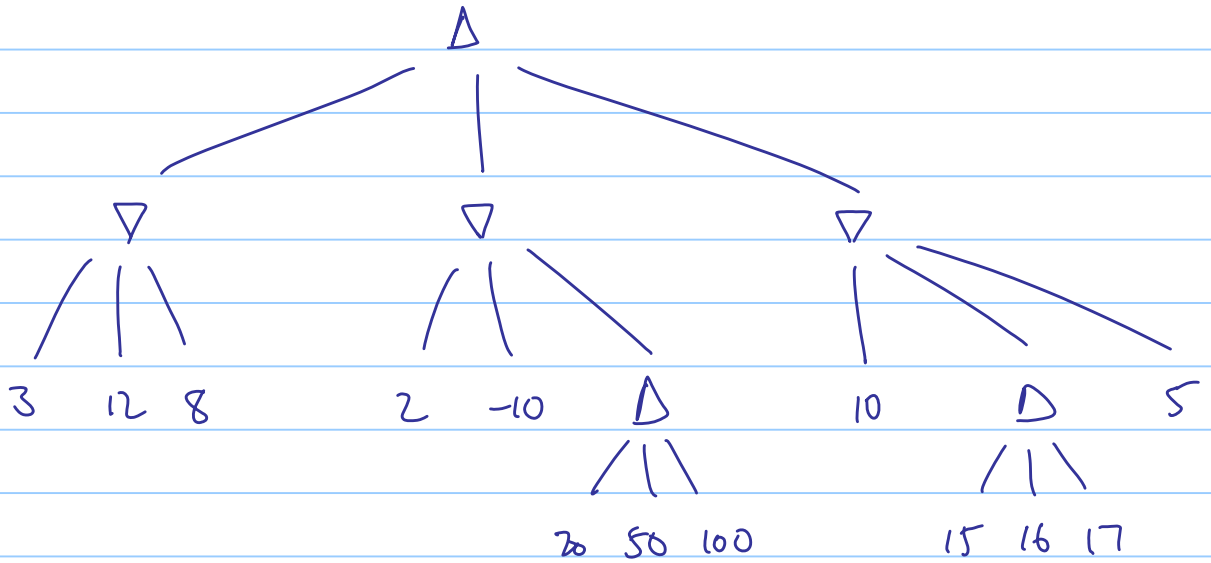
try to run minimax:

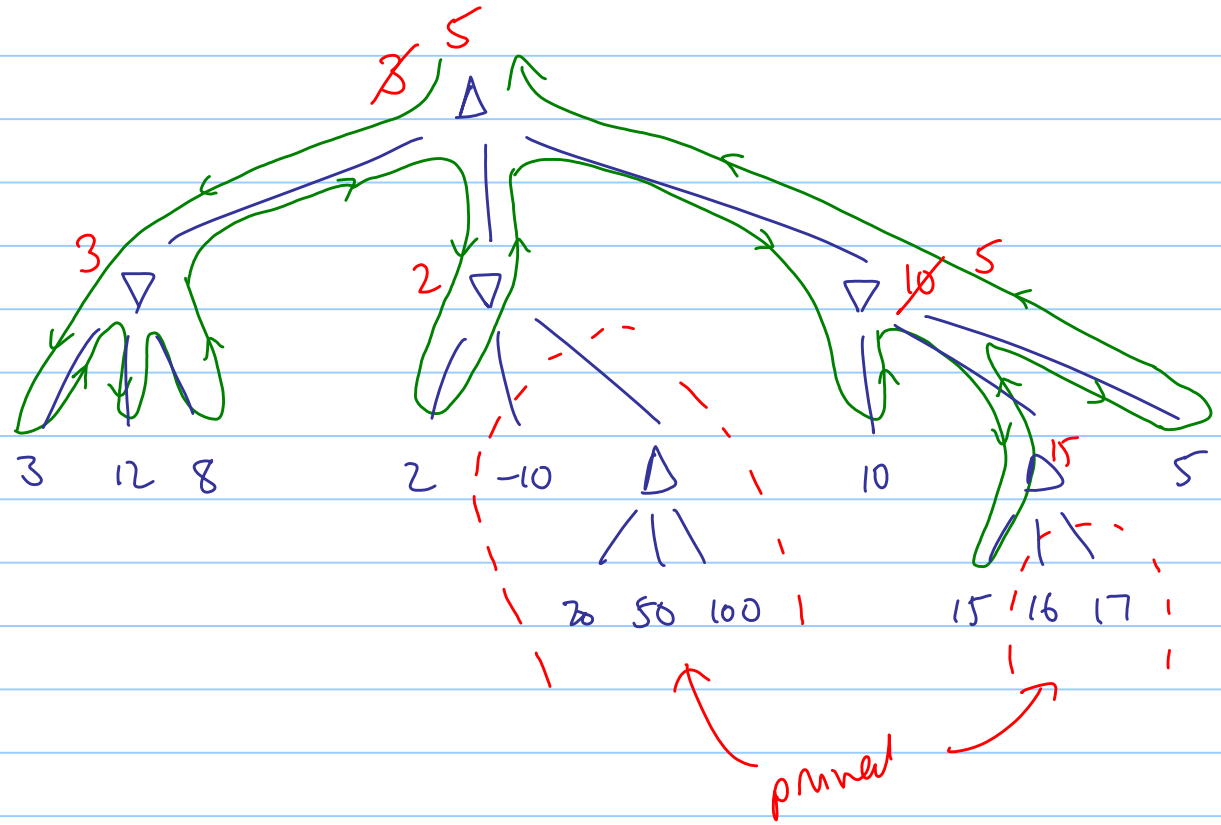


This is the idea of α-β pruning: based on knowledge of best strategy so far, can eliminate ("prune") some branches of the tree, without affecting correctness.

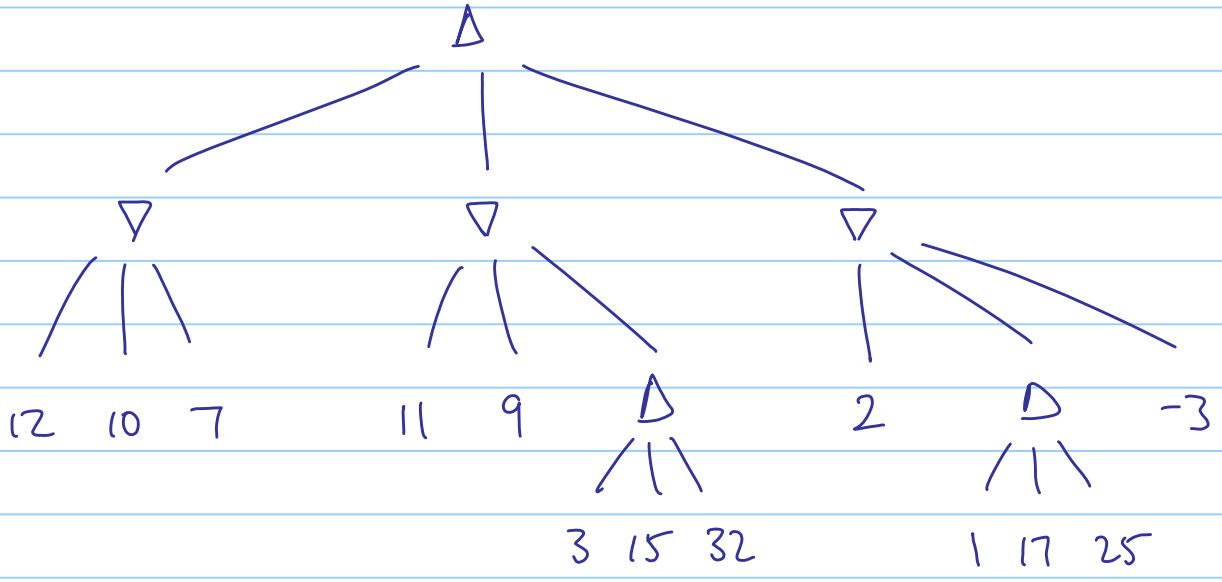
[α-β is a particular example of a general algorithmic approach known as "branch and bound"]

a) Informal α-β algorithm: explore depth-first, note best strategy so far, use common sense to prune.

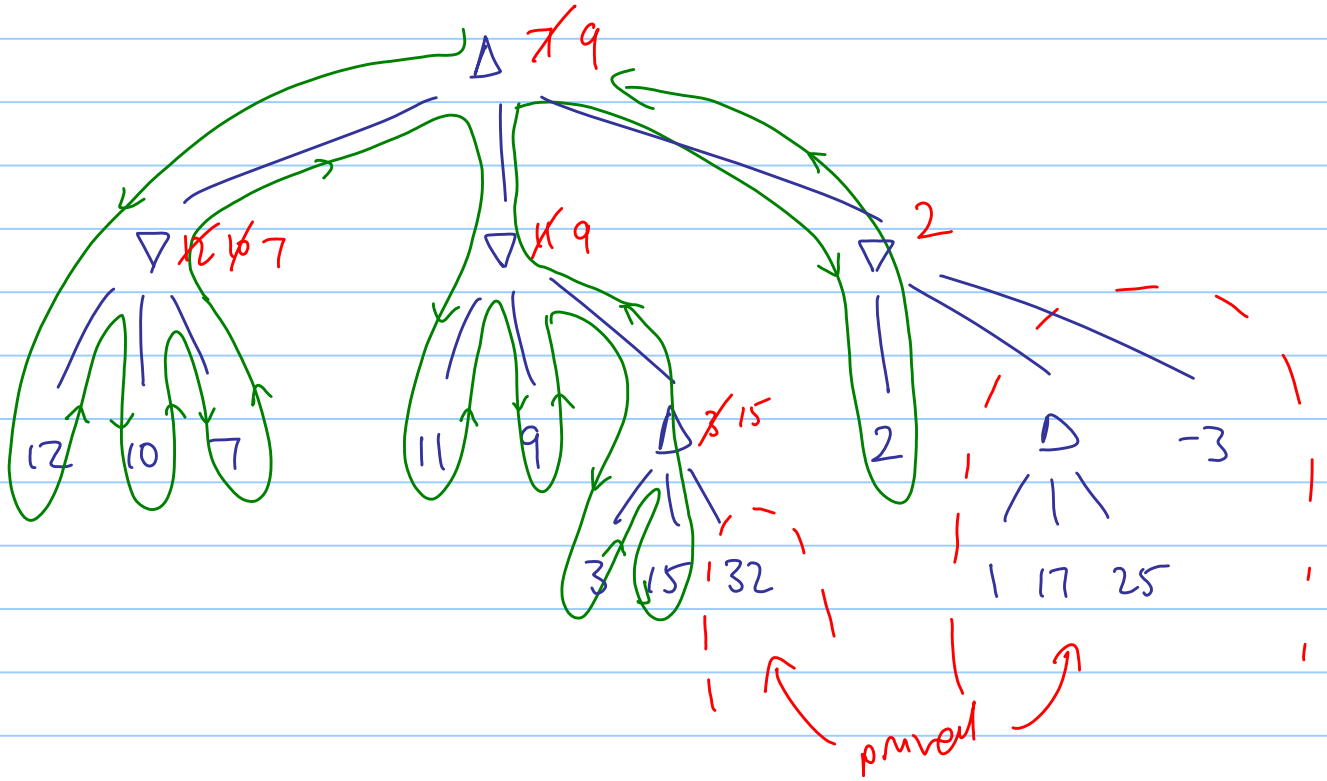




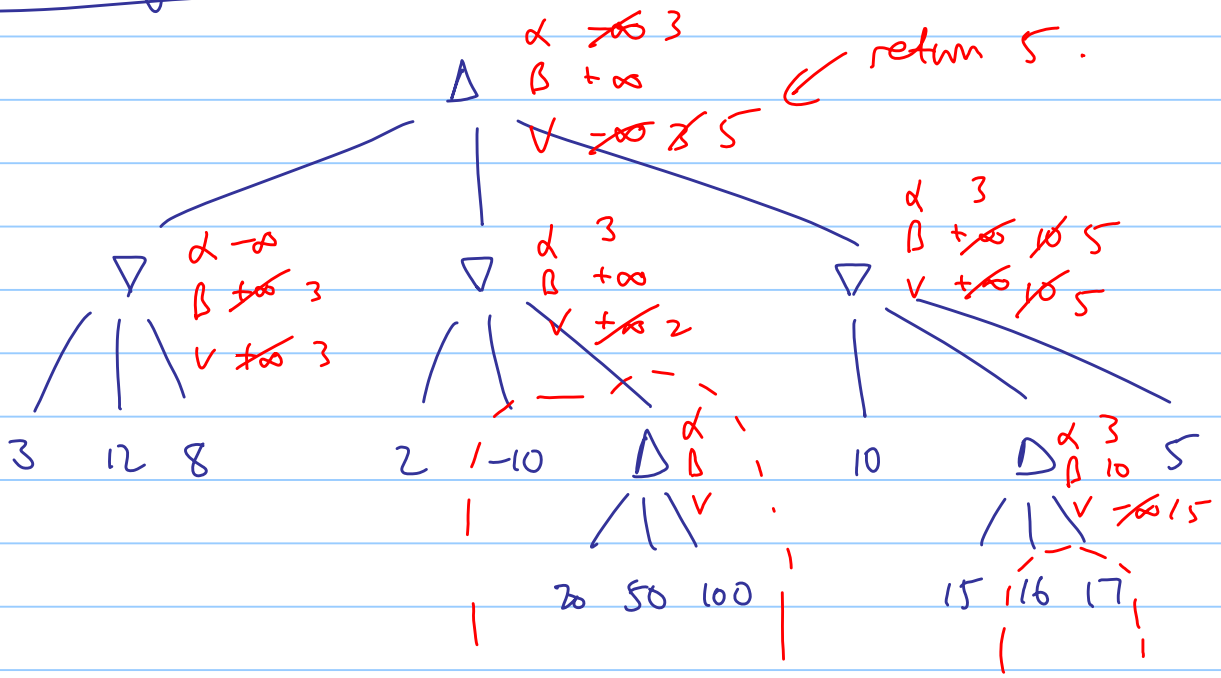
Another example to try informal algorithm:



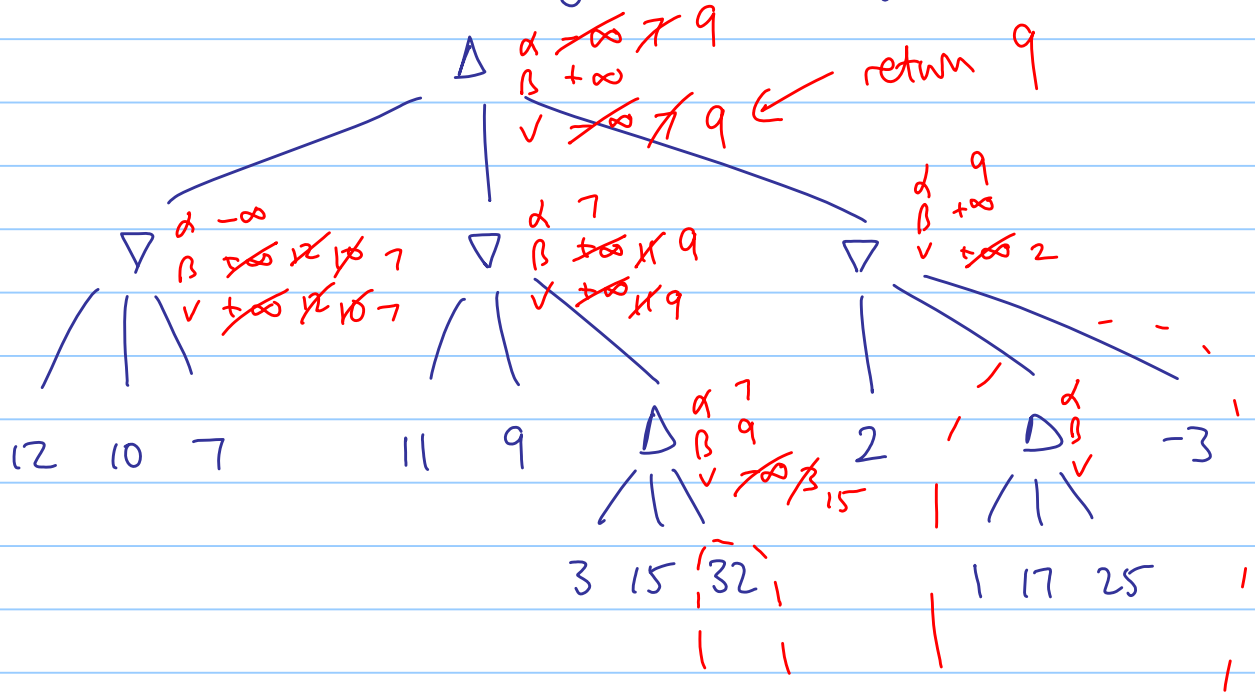
Answer:



b) Formal algorithm for α-β : See textbook fig 5-7



Another example to try formal algorithm:



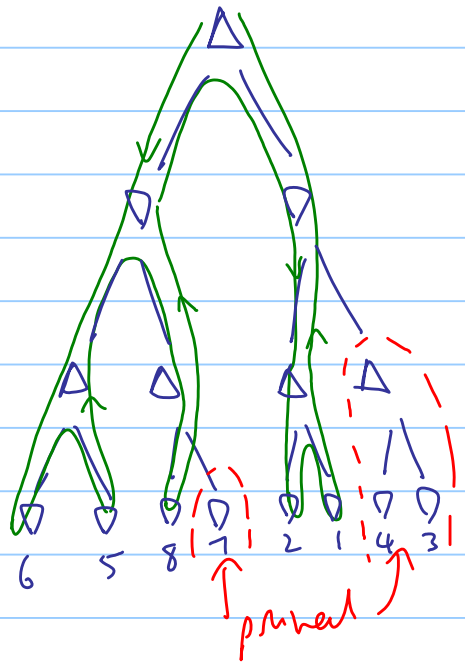
(3) Effect of move ordering on α-β

Note that we only get to prune if we consider some good move before some bad one.

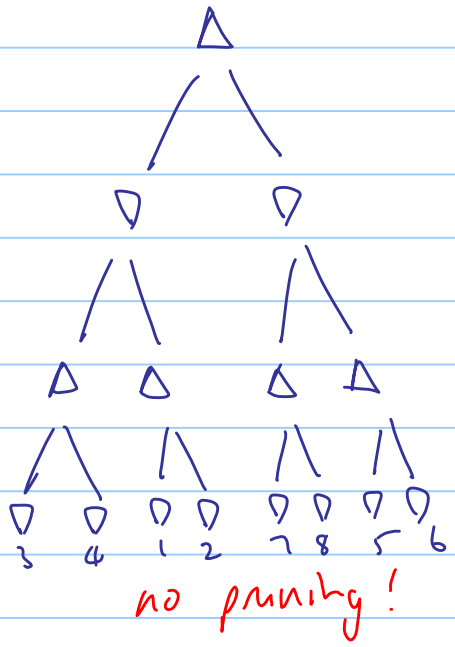
The ideal is to always consider moves in order from best to worst.

e.g.

best-to-worst



worst-to-best



(4) Complexity of α-β

- Recall that minimax is $O(b^m)$
- With optimal move ordering, α-β is $O(b^{m/2})$

  [equivalent of changing b to $\sqrt{b}$ or m to $m/2$
  ie. can explore twice as deep for the same cost]

- With random move ordering, α-β is approx $O(b^{3m/4})$
  (for moderate values of b)

(5) [Optional, for interest only.]

Check out the Knuth paper available on Moodle:

- skip over the math and just enjoy it as a superb piece of scientific writing
- look at intro to section 6 and the statement (but not proof) of Theorem 1. This tells you exactly which nodes are examined when the moves are ordered optimally.
- interesting quote, p304: "The α-β technique seems to be quite difficult to communicate verbally..."
- amusing and interesting passage on p316: "But as mentioned above, ..."