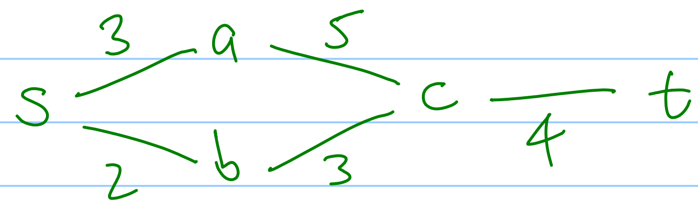
Lecture notes for AI class, "Heuristics for informed search"

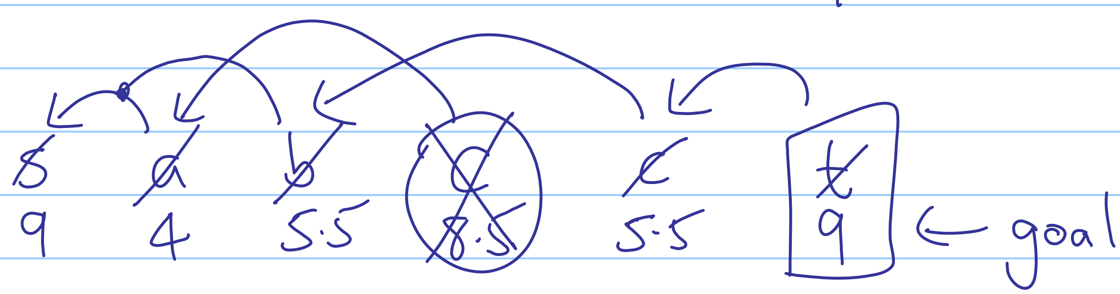
First, some simple examples to get us thinking.
Want to get from source s to target t on following map:



Example 1

Use heuristic:

n	h(n)
s	9
a	1
b	3.5
c	0.5

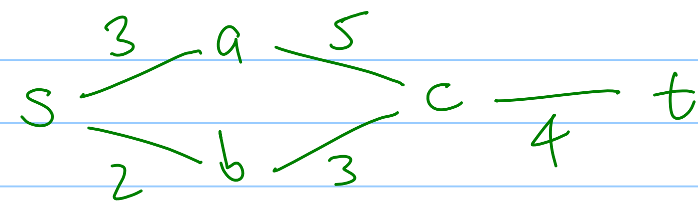


Note the need to <u>replace</u> c in the frontier.

(as described in text book – see last line of fig 3.14)

Example 2

Same map:



Different heuristic:

n	h(n)
s	9
a	4
b	7
c	0.5

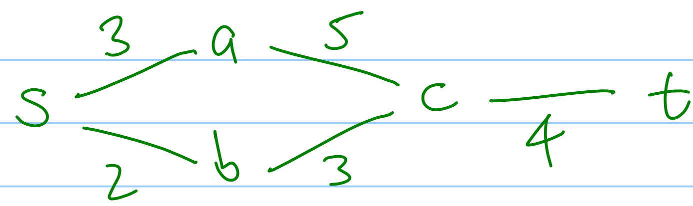
~~s~~	~~a~~	b	~~c~~	t
9	7	9	8.5	12

c is already explored, so nothing to expand. End up choosing nonoptimal s–a–c–t path!

Note: this would have worked if we used <u>tree</u> search rather than <u>graph</u> search.

Example 3

Same map:



Different heuristic:

n	h(n)
s	9
a	4
b	11
c	0.5

$\cancel{s}$	$\cancel{a}$	b	$\cancel{c}$	$\boxed{\cancel{t}}$	
9	7	13	8.5	$\boxed{12}$	← goal

Chose non-optimal s–a–c–t path!
Problem persists even with tree search!

What went wrong?
We got stuck at b – b wasn't expanded because its cost was overestimated.

<u>FACT</u>: If h never overestimates, A* <u>tree</u> search is <u>optimal</u>

this is called "admissible"

Definition If for all states s,

$$h(s) \leq \text{optimal cost to goal from } s$$

then h is **admissible**

e.g. straight-line distances on a map are admissible.

Fact A^* tree search is optimal for admissible heuristics.

Question: which of the earlier heuristics were admissible?

Definition If, for all pairs of states s, s' where s' is generated by applying some action to s, we have

$$h(s) \leq (\text{cost from } s \text{ to } s') + h(s')$$

then h is **consistent**.

e.g. straight-line distances on a map are consistent because of triangle inequality.

Fact A^* graph search is optimal for consistent heuristics.

Fact $\quad$ Consistent $\Rightarrow$ admissible

proof: Suppose h is consistent and let $s_0, s_1, s_2, \ldots s_n$ be an optimal path from s_0 to s_n.

Then

$$
\begin{aligned}
h(s_0) &\le \text{cost}(s_0, s_1) + h(s_1) \\
&\le \text{cost}(s_0, s_1) + \text{cost}(s_1, s_2) + h(s_2) \\
&\;\vdots \\
&\le \sum_{i=0}^{n-1} \text{cost}(s_i, s_{i+1}) \\
&= \text{optimal cost to goal.}
\end{aligned}
$$

by defn of consistency

QED. $\square$