

stochastic games

Note Title

Review of expected value from basic probability:

Given some numerical outcomes and probabilities for those outcomes, the expected value is the sum of prob \times outcome.

e.g. • one dice:

	1	2	3	4	5	6
prob	$\frac{1}{6}$	$\frac{1}{6}$	---	---	---	$\frac{1}{6}$

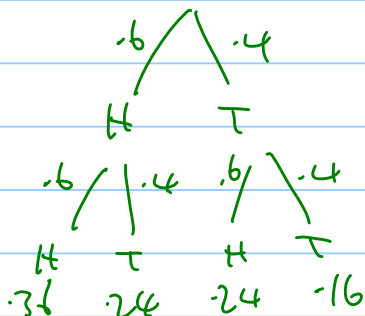
$$\text{exp. val.} = \left(\frac{1}{6} \times 1\right) + \left(\frac{1}{6} \times 2\right) + \dots + \left(\frac{1}{6} \times 6\right) = 3.5$$

• two dice:

	2	3	4	---	11	12
	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	---	$\frac{2}{36}$	$\frac{1}{36}$

$$\text{exp. val.} = \left(\frac{1}{36} \times 2\right) + \left(\frac{2}{36} \times 3\right) + \dots + \left(\frac{1}{36} \times 12\right) = 7$$

Exercise: 2 biased coins. Chance of heads is 0.6 for each. Both are flipped, you win \$3 for each head showing. What are your expected winnings?



num heads	0	1	2
prob	.16	.48	.36
outcome	0	3	6

$$\begin{aligned} \text{exp. val.} &= 0 \times .16 + 3 \times .48 + 6 \times .36 \\ &= 3.6 \end{aligned}$$

Solving a stochastic game:

- (1) Use chance nodes (as well as MAX, MIN)
- (2) Use minimax, but compute expected value of minimax at chance nodes
e.g. Russell slide 26, Try it with biased coin.
and see formal algorithm on p148.
- (3) Can cut off and evaluate if desired.

Notes:

- complexity is horrible: $O(b^m \sqrt[n]{b^m})$ number of distinct chance outcomes
- pruning like α - β is possible but tricky - see homework exercise 5.16.
- can instead evaluate using Monte Carlo simulation...

Monte Carlo simulation

Basic idea: at chance node, do not evaluate all possibilities. Instead, pick a random sample of the outcome, compute minimax for each, and return the average. See p180 and p184 for a few more details.

Final note: The ideas in this lecture contain some promising material for the final project of the course. :)