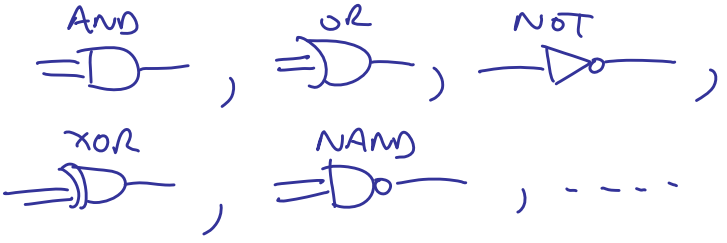
# Combinational circuits

Last time:
- Boolean functions e.g. $f(a,b,c) = ab + a\bar{c} + \bar{b}c$

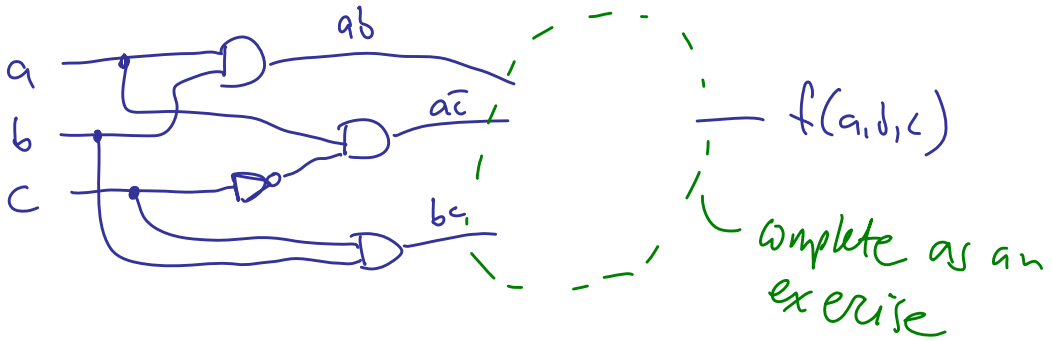
truth table:

| a | b | c | ab | $a\bar{c}$ | $\bar{b}c$ | f |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | | | | |
| 0 | 0 | 1 | | | | |
| 0 | 1 | 0 | | | | |
| 0 | 1 | 1 | | | | |
| 1 | 0 | 0 | | | | |
| 1 | 0 | 1 | | | | |
| 1 | 1 | 0 | | | | |
| 1 | 1 | 1 | | | | |

fill in as exercise

- Logic gates e.g. 


- Boolean functions as circuits



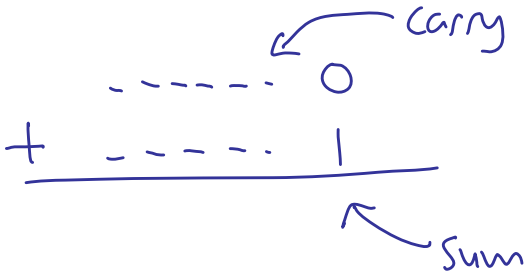

Demo: TruthTable.java with $a + \bar{b} + \bar{c}$

Today: Combinatorial circuits

- they produce outputs (almost instantaneously) from present inputs (do not depend on past inputs)
- any boolean function can be implemented as a combinatorial circuit
- every combinatorial circuit computes a boolean function.
- we study specific, useful circuits:
  1. half adder
  2. full adder
  3. decoder
  4. multiplexer

# ① Half adder

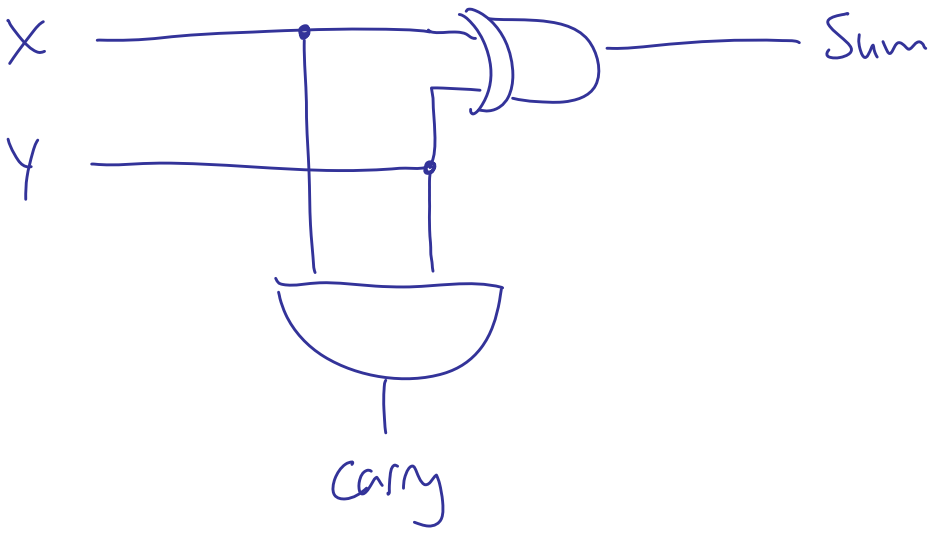
motivation: computes last digit of binary add




| Inputs | | Outputs | |
|---|---|---|---|
| X | Y | Sum | Carry |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

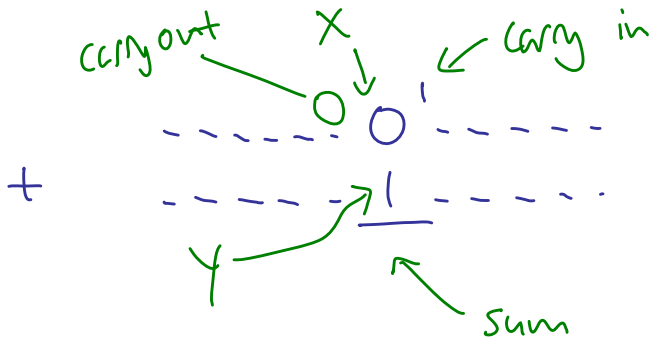
single gate? XOR

single gate? AND

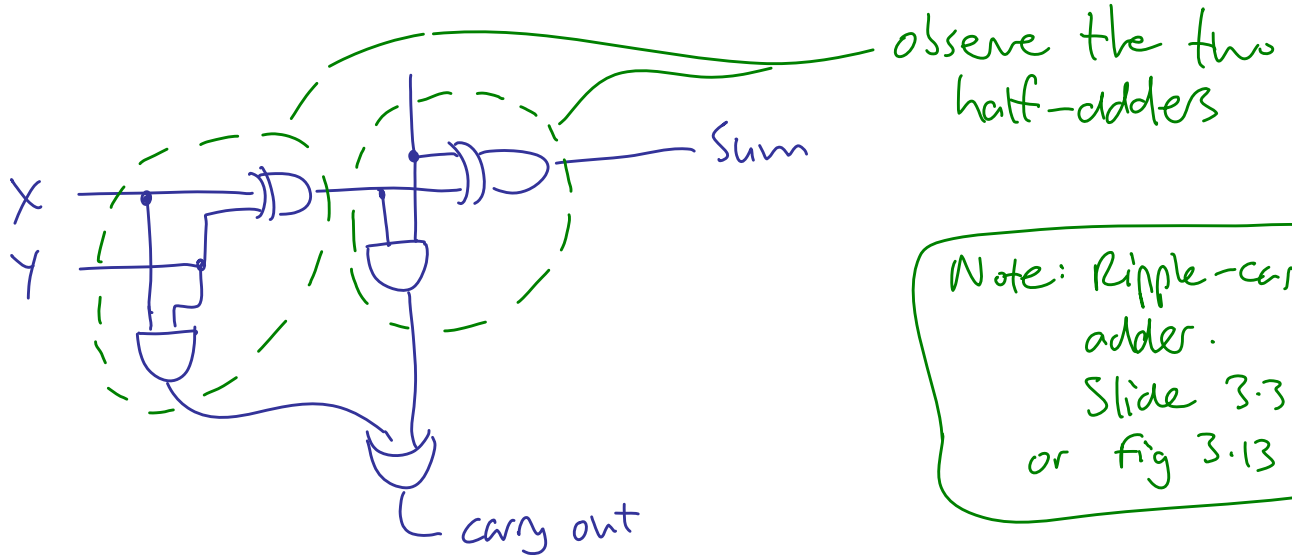


## (2) Full adder

motivation : middle digit of binary add




| Inputs | | | Outputs | |
|---|---|---|---|---|
| X | Y | Carry In | Sum | Carry out |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

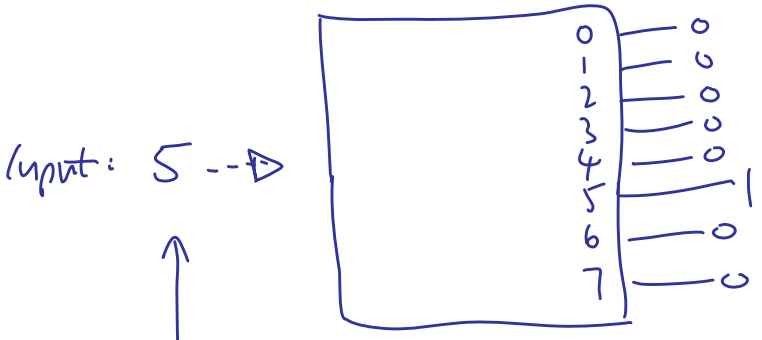



Note: Ripple-carry adder.
Slide 3.35
or fig 3.13

# ③ Decoder

- Input describes a number in some range, eg. $n \in \{0, 1, 2, \ldots 63\}$
- Output has one line for each possible input
- When input is $n$, the $n$th output line is on, all others are off

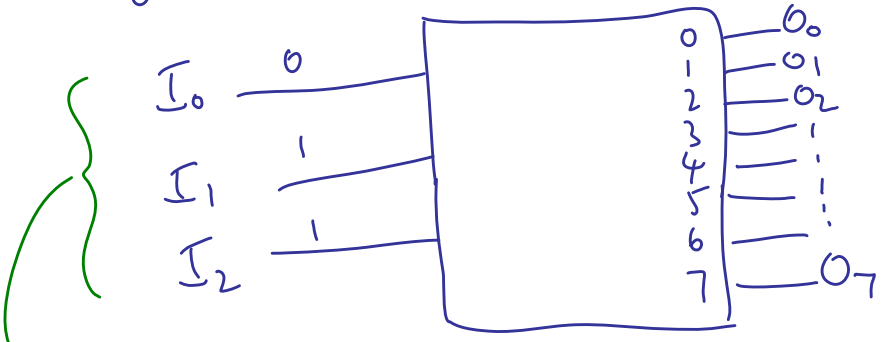
Motivation: Useful for activating a memory location based on its numerical address




how to describe this input?

Represent $n$ as unsigned int. Need 3 input lines

e.g. $n = 6$

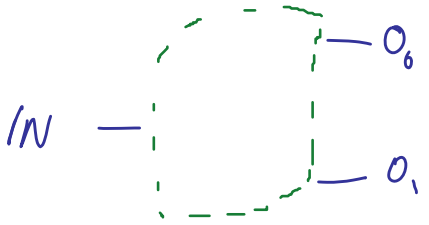



note ordering of bits and interpretation. See fig 3.15 in textbook

Activity: Solve decoder puzzle on resources page.

ACTIVITY: create a 1-to-2 decoder

Answer:

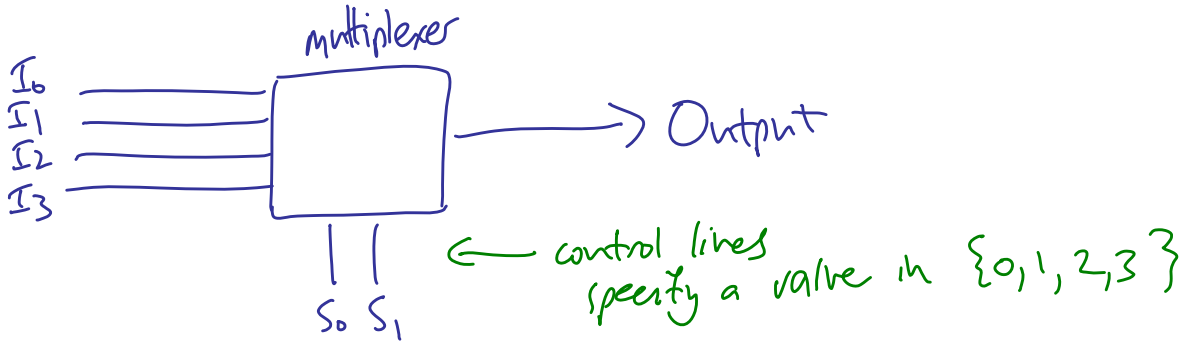



(Fill in missing part)

## ④ Multiplexer

- control lines specify a number in some range
  e.g. $n \in \{0, 1, \ldots 31\}$
- input has one line for each control value.
- the single output is a copy of the nth input

useful for e.g. selecting a desired input on a data projector.




Activity: solve the multiplexer puzzle on resources page

See also: bit shifter circuit (consult textbook & puzzles handout)

Objective is to shift the input left or right

e.g. 1101 shifted left is 1010 (fill in with 0)
shifted right is 0110 (fill in with 0)

Activity: create a 2-to-1 multiplexer in SimCir.

Solution:


(fill in missing part)

Challenge: Build a ripple carry adder for 4-bit addition.

Also, see online demos on resources page.