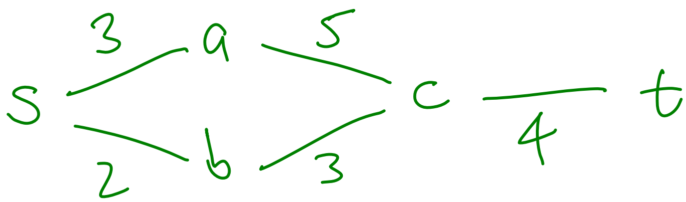
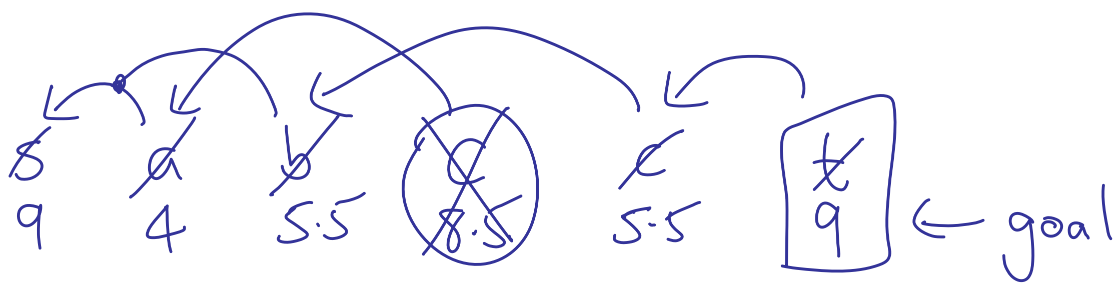
Lecture notes for AI class, "Heuristics for informed search"

First, some simple examples to get us thinking.
Want to get from source s to target t on following map:



__Example 1__ Use heuristic:

n	h(n)
s	9
a	1
b	3.5
c	0.5

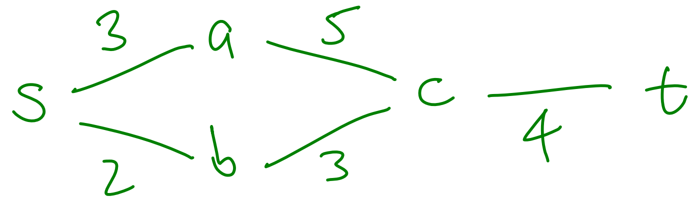


Note the need to replace c in the frontier.

(as described in text book – see last line of fig 3.14)

Example 2

Same map:



Different heuristic:

n	h(n)
s	9
a	4
b	7
c	0.5

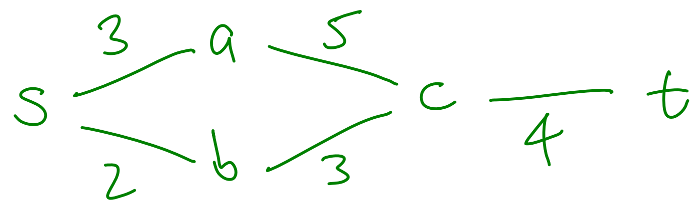
~~s~~	~~a~~	b	~~c~~	t
9	7	9	8.5	12

c is already explored, so nothing to expand. End up choosing nonoptimal s–a–c–t path!

Note: this would have worked if we used <u>tree</u> search rather than <u>graph</u> search.

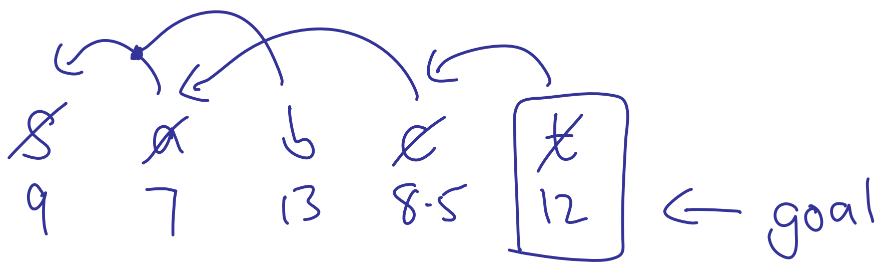
Example 3

Same map:



Different heuristic:

n	$h(n)$
s	9
a	4
b	11
c	0.5



Chose non-optimal s-a-c-t path!
Problem persists even with tree search!

What went wrong?

We got stuck at b – b wasn't expanded because its cost was overestimated.

FACT: If h never overestimates, A^* tree search is optimal

this is called "admissible"

Definition If for all states s,

$$h(s) \leq \text{optimal cost to goal from } s$$

then h is admissible

e.g. straight-line distances on a map are admissible.

Fact A^* tree search is optimal for admissible heuristics.

Question: which of the earlier heuristics were admissible?

Definition If, for all pairs of states s, s' where s' is generated by applying some action to s, we have

$$h(s) \leq (\text{cost from } s \text{ to } s') + h(s')$$

then h is consistent.

e.g. straight-line distances on a map are consistent because of triangle inequality.

Fact A^* graph search is optimal for consistent heuristics

<u>Fact</u> Consistent $\Rightarrow$ admissible

proof: Suppose h is consistent and let $S_0, S_1, S_2, \dots S_n$ be an optimal path from S_0 to S_n.

Then

$$
\begin{aligned}
h(S_0) &\leq \operatorname{cost}(S_0, S_1) + h(S_1) \\
&\leq \operatorname{cost}(S_0, S_1) + \operatorname{cost}(S_1, S_2) + h(S_2) \\
&\ \vdots \\
&\leq \sum_{i=0}^{n-1} \operatorname{cost}(S_i, S_{i+1}) \\
&= \text{optimal cost to goal.}
\end{aligned}
$$

by defn of consistency

QED. $\square$