First-order logic

1. Motivation

PL (propositional logic) is inefficient at representing certain commonly-occurring concepts.

e.g. (a) Describing the fact that all the students in this class are CS majors

we need atoms: P_1: Tim is a CS major
                 ⋮
                 P_q: Sasha is a CS major

KB is: $P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_q$

Easier is: $\forall x \; CSMajor(x)$
                    ↖ "for all"

(b) Describe "Someone in the class is an Astronomy minor"

we need atoms: Q_1: Tim is an Astronomy minor
                 ⋮
                 Q_q: Sasha is an Astronomy minor

KB is $Q_1 \vee Q_2 \vee \dots \vee Q_q$.
Easier is: $\exists x \; AstronomyMinor(x)$
                  ↖ there exists

Thus, the basic idea behind **First-Order Logic** (FOL) is to add quantifiers $(\forall, \exists)$ to propositional logic.

2. Details on first order logic

In more detail, FOL uses:

- **objects** — the elements in the domain of discourse (e.g. students in this class)

- **relations** — basic statements about the objects that

  (note: The textbook calls these **predicates**.)

  can be true or false
  e.g. CSMajor (Ashir)
       Helped (Joanna, Lam)
       Team (Tucker, Vy, Jon)

- **functions** — input is one or more objects, output is an object

  e.g.   Prog AsstPartner (Lam)
                              evaluates to Sasha

- **quantifiers** — $\forall, \exists$.

More details on relations:

- a unary relation is a <u>property</u>

    e.g. MathMajor(Tim) means "Tim is a math major"
    Blue(Sky) means "The sky is blue"

- binary relations follow an important ordering convention:
    $P(X,Y)$ often means "X is a P of Y"
                        or   "X P Y"

    e.g. Instructor($John$, $Sasha$) means
                        "John is the instructor of Sasha"
    Helped(Vy, $Tucker$) means
                        "Vy helped Tucker"

<u>BIG WARNING</u>: Functions and relations are completely
                    different, but look the same.

    Functions return <u>objects</u>
    Relations return <u>true or false</u>

Examples:

1. Father($John$) means:

    fill in as
    exercise ⟶

relation	function

Son (Leslie, Rowan) means

fill in as exercise →

relation	function

De Morgan's rules for quantifiers

- $\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$

  e.g. not all students in this class are CS majors
  $\equiv$
  fill in as exercise

- $\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$

  e.g. no student in the class is an Astronomy Minor
  $\equiv$
  fill in as exercise

Note: follows easily from PL version of De Morgan, since $\forall$ is like a conjunction and $\exists$ is like a disjunction.

Meaning of equals

The $(=)$ sign means "the same object"

e.g. ProgAsstPartner (Sasha) = Lam

3. examples

Important examples from text book: 8.10, 8.11, 8.24

for interest only: see real-world applications of FOL
on resources web page.