Highway Dimension and Provably Efficient Shortest Path Algorithms

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(Joint work with Ittai Abraham, Amos Fiat, and Renato Werneck)

Motivation: Computing Driving Directions

Outline

Theory vs. Practice

[Anonymous]

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This talk
How and why modern routing algorithms work.

Recent Developments

Continent-sized road networks have 10s of millions intersections.

Dijkstra's algorithm: ≈ 5 s

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- Arc flags [Lauther 04, Köhler et al. 06].
- A^* with landmarks [Goldberg & Harrelson 05].
- Reach [Gutman 04, Goldberg et al. 06].
- Highway hierarchies [Sanders & Schultes 05].
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- DIMACS Shortest Paths Implementation Challenge (2005–2006).

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Greatly improved performance: < 1 ms, ≈ 0.1 s on a mobile device.
Only a few hundred intersections searched.

Definitions and Model

Input

- Graph $G = (V, E)$ (intersections, road segments), $|V| = n$, $|E| = m$.
- Weight function ℓ (length, transit time, fuel consumption, ...).
- Static problem, G and ℓ incorporate all modeling information.

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Algorithms with preprocessing

- Two phases: practical preprocessing and real-time queries.
- Preprocessing output not much bigger than the input.
- Preprocessing may use more resources than queries.

Dijkstra's Algorithm

[Dijkstra 1959], [Dantzig 1963].

Dijkstra's Algorithm

- Examine vertices in the order of their distance from s.
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- Run algorithm from t in the graph with all arcs reversed.
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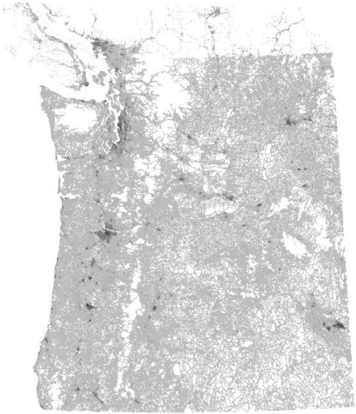
Bidirectional Algorithm

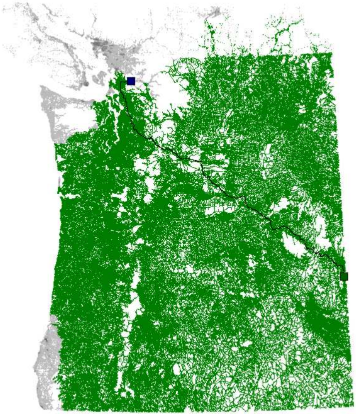
- Run forward Dijkstra from s and backward from t.
- Stop when the searches meet.

Example Graph



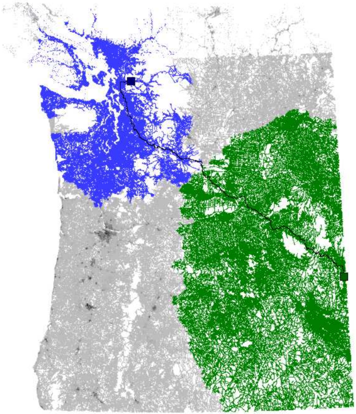
1.6M vertices, 3.8M arcs, travel time metric.

Dijkstra's Algorithm



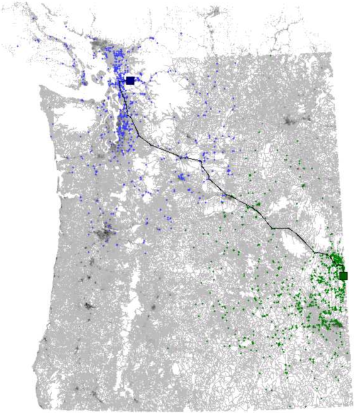
Searched area

Bidirectional Algorithm



forward search/ **reverse search**

Reach Algorithm



Pruning leads to amazing speedup.

Three Recent Algorithms

Algorithm intuition

- Reach pruning (RE): Local intersections far from origin/destination can be ignored.

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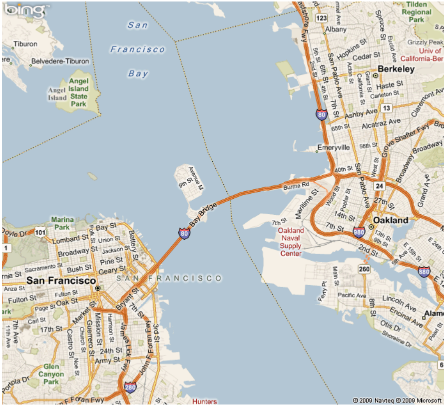
These intuitive ideas can be mathematically formalized and lead to provably correct algorithms which work very well on road networks.

Reach Fundamentals



[Gutman 04; Goldberg et al. 06]

Preprocessing computes intersection locality.

Query uses locality to prune search.

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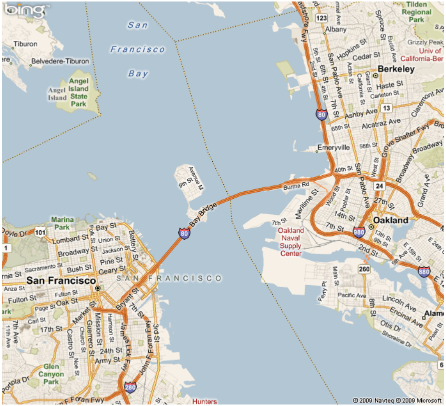

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- Consider a vertex v that splits a path P into P_1 and P_2. $r_P(v) = \min(\ell(P_1), \ell(P_2))$.

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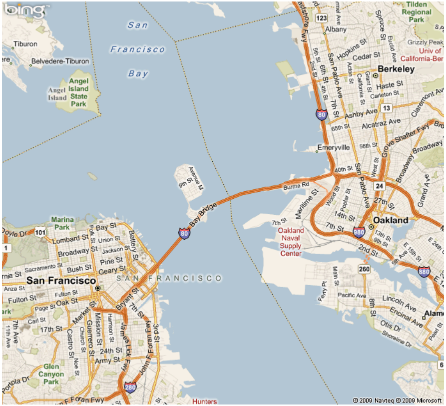

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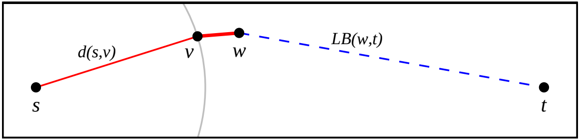
Definition of reach

- Consider a vertex v that splits a path P into P_1 and P_2. $r_P(v) = \min(\ell(P_1), \ell(P_2))$.
- $r(v) = \max_P(r_P(v))$ over all shortest paths P through v.

Pruning Search Using Reach



Pruning

If $r(w) < \min(d(v) + \ell(v, w), LB(w, t))$ then prune w.

Pruning Search Using Reach

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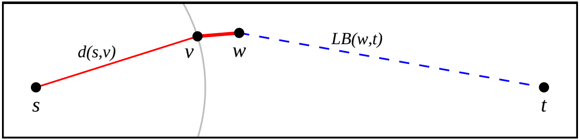

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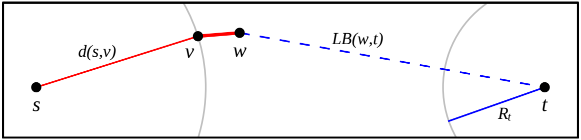
Lower bounds for nothing



Bidirectional search gives implicit bounds (R_t below).

RE Algorithm

RE Query

- Bidirectional Dijkstra's algorithm with pruning based on reaches.
- A small change to Dijkstra's algorithm.

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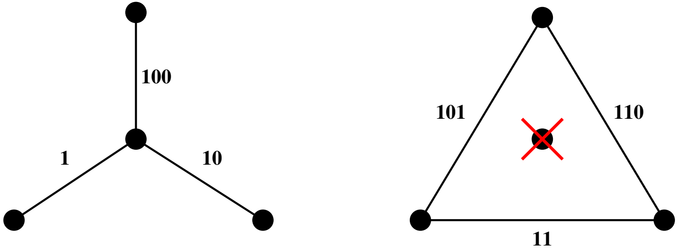
Remarks

- $O(nm)$ perprocessing impractical on large graphs.
- Fast heuristic preprocessing computes reach upper bounds.
- Shortcuts speed up both preprocessing and query.
- CH algorithm shows that shortcuts are crucial.

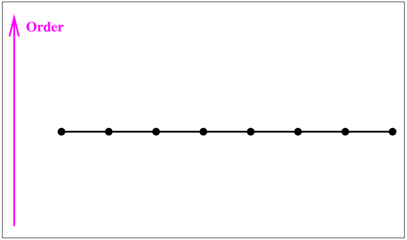
Shortcut Operation

[Sanders & Schultes 05]
The key operation for Contraction Hierarchies algorithm

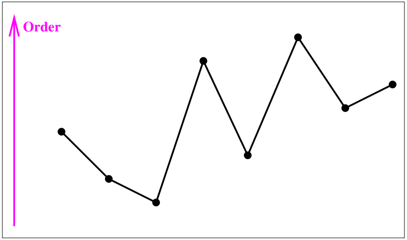


A shortcut arc can be omitted if redundant (alternative path exists).

Contraction Hierarchies



[Geisberger et al. 08]
Preprocessing orders vertices, order corresponds to locality. Both forward and reverse searches consider only “up” (more local to more global) edges. Effective pruning.



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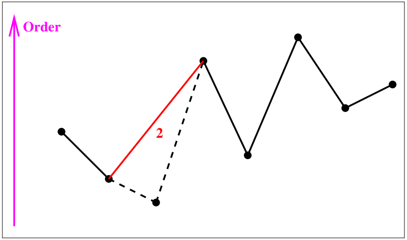

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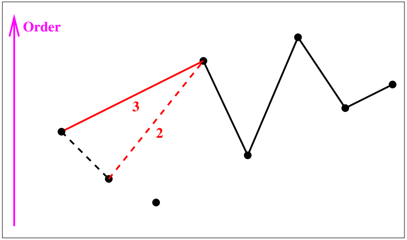

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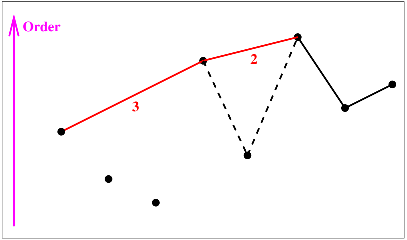

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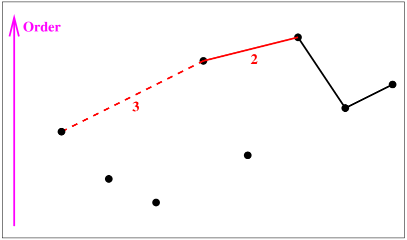

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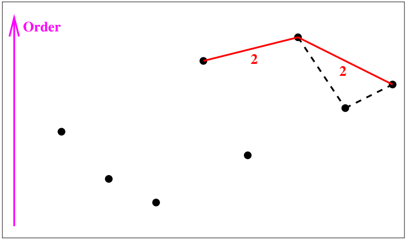

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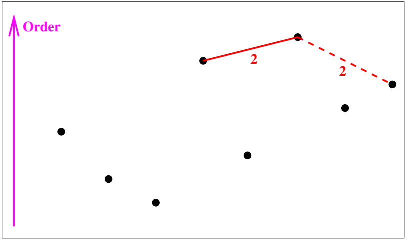

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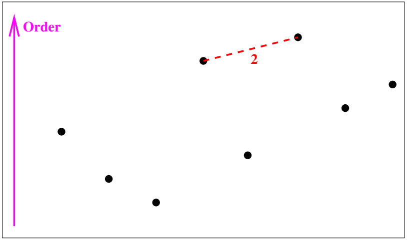

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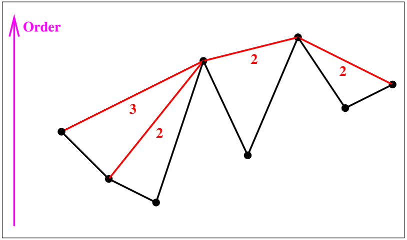

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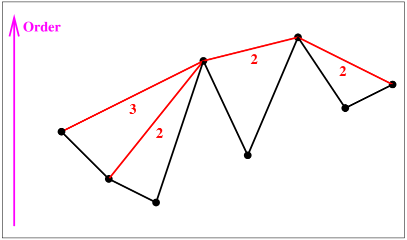

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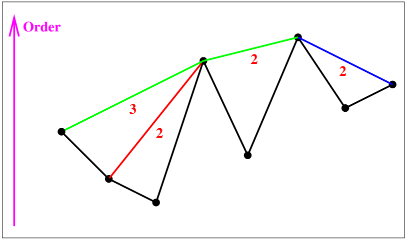
- Run a modified bidirectional Dijkstra's algorithm.



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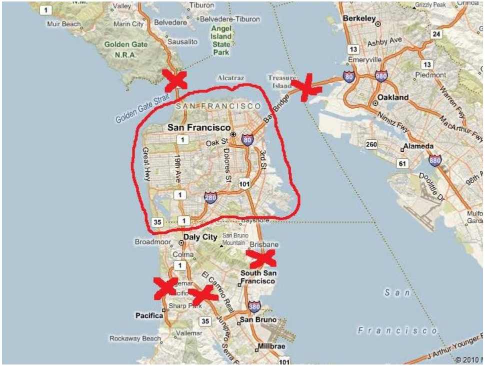
Query algorithm

- Run a modified bidirectional Dijkstra's algorithm.
- The searches only consider "up" edges.

Transit Node Intuition



For a region, there is a small set of nodes such that all sufficiently long shortest paths out of the region pass a node in the set.

TN Preprocessing

[Bast et al. 06]

Basic concepts

- Divide a map into regions (a few thousand).
- For each region, optimal paths to far away places pass through one of a small number of access nodes (≈ 10 on the average).
- The union of access nodes is the set of transit nodes ($\approx 10\,000$).

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Preprocessing Algorithm

- Find access nodes for every region.
- Connect each vertex to its access nodes.
- Compute all pairs of shortest paths between transit nodes.

TN Query

Long-range query algorithm

- The shortest path has the form s – access(s) – access(t) – t

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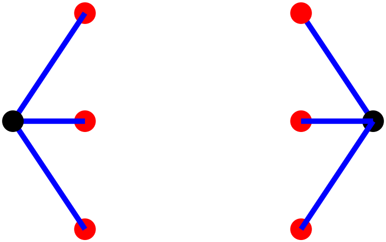

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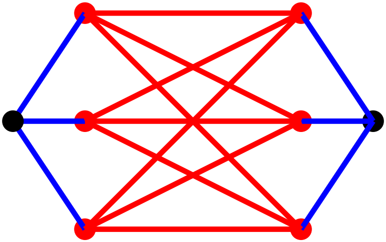
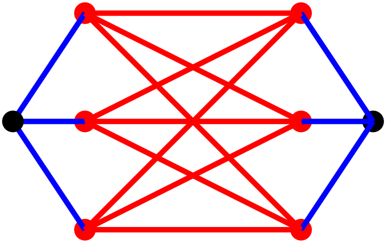


TN Query

Long-range query algorithm

- The shortest path has the form s – access(s) – access(t) – t
- Table look-up for the (access(s), access(t)) node pairs.



Remarks

- Very fast: 10×10 table look-ups per long-range query.
- Local queries: another method or hierarchical approach.

Theoretical Results

Practice

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- Why do they work well on road networks?
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Theory [Abraham, Fiat, Goldberg & Werneck 10]

- Define highway dimension (HD).
- Good time bounds for the three algorithms assuming HD is small.
- Analysis highlights algorithm similarities.
- Generative model of small HD networks (road network formation).

In the spirit of the small world model [Milgram 67, Kleinberg 99].

Definitions and Remarks

Definitions and assumptions

- Constant maximum degree.
- $B_{v,r}$ denotes the set of vertices within distance r from v.
- $|P|$ denotes the length of P.
- h denotes highway dimension.
- k denotes either h or $O(h \log n)$ (exponential or poly-time preprocessing).
- Network diameter D.

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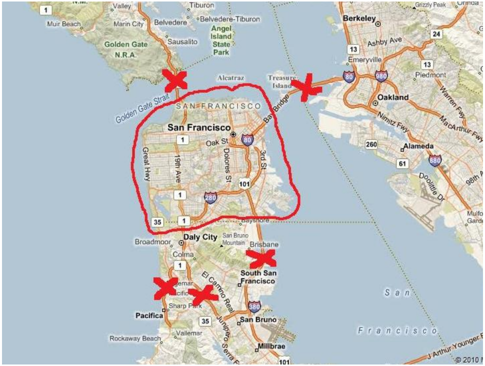
Remarks

- HD definition motivated by Transit Nodes.
- Preprocessing based on Contraction Hierarchies ideas.
- Analysis based on Reach ideas.

Highway Dimension Motivation



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Highway Dimension Definition

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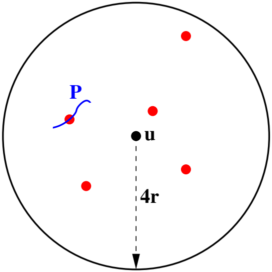

Highway dimension (HD) h

$$\forall \quad r \in \Re, \forall u \in V, \exists S \subseteq B_{u,4r}, |S| \leq h, \text{ such that}$$

$$\forall \quad v, w \in B_{u,4r},$$

if P is a SP: $\ell(P(v,w)) > r$ and $P(v,w) \subseteq B_{u,4r}$,

then $P(v,w) \cap S \neq \emptyset$.

Highway vs. Doubling Dimension

Definition

A metric space has a doubling dimension α if every ball of radius r can be covered by 2^α balls of radius $r/2$.

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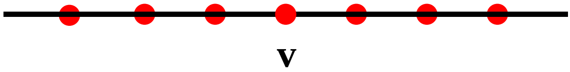
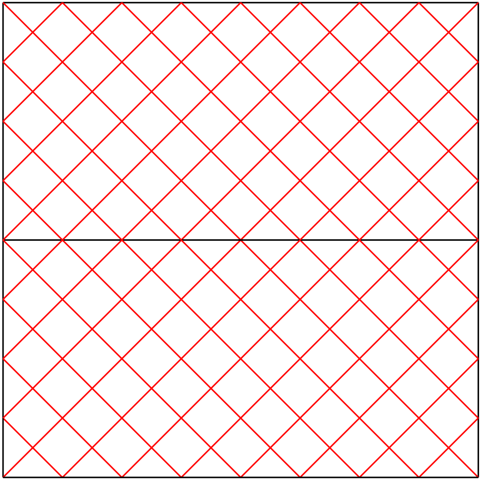
Line: Small HD



A line has HD 7 and doubling dimension 1.

Grid: High HD



A grid has HD $\Theta(\sqrt{n})$ and the doubling dimension 2.

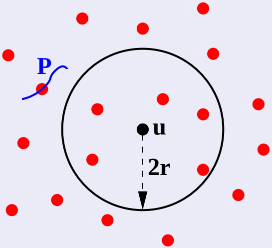
Shortest Path Covers

All SPs in a range can be covered by a sparse set.

(r, k) Shortest path cover ((r, k)-SPC):

A set C such that

$$\forall \quad \text{SP } P: \; r < |P| \leq 2r \Rightarrow$$
$$P \cap C \neq \emptyset \quad \text{and}$$
$$\forall u \in V, \; |C \cap B_{u,2r}| \leq k$$



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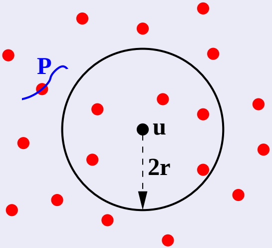
(r,k) Shortest path cover ((r,k)-SPC):

A set C such that

$$\forall \quad \text{SP } P: \; r < |P| \leq 2r \Rightarrow$$
$$P \cap C \neq \emptyset \quad \text{and}$$
$$\forall u \in V, \; |C \cap B_{u,2r}| \leq k$$



Can use constants different from 4 and 2, but the constants are related.

HD vs. SPC

Theorem

If G has highway dimension h, then $\forall\ r\ \exists$ an (r,h)-SPC.

Proof idea: Show that S^*, the smallest set that covers all shortest paths $P:\ r < |P| \leq 2r$, is an (r,h)-SPC.

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Proofs depend on the choice of constants in the definitions.

Generic Preprocessing

Preprocessing algorithm

- Let $S_0 = V$. For $1 \le i \le \log D$ build $(2^i, k)$-SPC covers S_i.
- Let $L_i = S_i - \bigcup_{i+1}^{\log D} S_j$ (vertex partitioning into layers).
- Order vertices so that L_i comes before L_{i+1}; ordering inside layers is arbitrary.
- Do shortcutting in this order to get E^+.

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Running time

Preprocessing is exponential ($k = h$) or polynomial ($k = O(h \log n)$).

Preprocessing Space

Lemma

For $v \in L_i$, $j \geq i$, the number of $(v, w) \in E^+$ with $w \in L_j$ is at most k.

Proof. (v, w) corresponds to P with internal vertices less than v, w. Thus $w \in B_{v,2 \cdot 2^i}$. The SPC definition implies the lemma.

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Things are better than the worst case in practice.

RE Preprocessing

Remarks

- Reach bounds are in the graph with shortcuts.
- Break ties based on hop count (prefer shortcuts).

Lemma

If $v \in L_i$ *then* $\text{reach}(v) \leq 2 \cdot 2^i$.

Proof: Suppose the reach is greater. Then there is a shortest path P that v divides into P_1 and P_2 with $|P_1|, |P_2| > 2 \cdot 2^i$. Both P_1 and P_2 contain vertices in L_j with $j > i$, so there is a shortcut from P_1 to P_2. But then P is not a shortest path.

Additional work is linear.

Query Time Bounds

Theorem

RE query takes $O((k \log D)^2)$ *time.*

Proof: Consider a forward search from s. In $B_{s,2 \cdot 2^i}$, the search scans only vertices of L_i in $B_{s,2 \cdot 2^i}$. Thus $O(k \log D)$ scans.

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Remarks

- Shortest path can be extracted in time linear in the number of its arcs.
- Similar analysis for CH yields the same bound.
- Also develop a faster version of TN: $O(k \log D)$ query.

Network Formation

Natural networks with constant highway dimension?

Attempt to model road networks

- Build on the Earth surface (low doubling dimension).
- Build in decentralized and incremental manner.
- Highways are faster than local roads.

Capture some, but not all, real-life properties.

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Setting

- Metric space (M, dist), doubling dim. $\log \alpha$, diameter D.
- Speedup parameter $\delta \in (0, 1)$.
- Edge $\{v, w\}$ has length $\text{dist}(v, w)$ and transit time $\tau(v, w) = \text{dist}^{1-\delta}(v, w)$. (Long roads are fast.)
- On-line network formation, adversarial vertex placement.

Network Formation (cont.)

In the spirit of dynamic spanners [Gottlieb & Roditty 08].

- Adversary adds vertices, we connect them.
- Intuition: connect a new village to all nearby villages and to the closest town.
- Formally: maintain covers C_i for $0 \leq i \leq \log D$.
  $C_0 = V$, $C_{i+1} \subseteq C_i$, vertices in C_i are at least 2^i apart.
- When adding a new vertex v, add v to $C_0, \ldots, C_i$ for appropriate i. (The first vertex added to all C's.)
- For $0 \leq j \leq i$, connect v to $C_j \cap B_{v,6\cdot 2^j}$.
- If $i < \log D$, connect v to the closest element of C_{i+1}.

Theorem

The network has highway dimension of $\alpha^{O(1)}$.

Final Remarks

Summary

- Intuitive and practical routing algorithms.
- Efficient implementations.
- Used in practice.
- Theoretical understanding and justification.
- Further research, e.g., improved bounds or algorithms for other problems assuming small HD (TSP, vehicle routing, etc.).
- Static problem solved, dynamic – active research area.
  - Real time traffic.
  - Historical data.
  - Combination (prediction).

Thank You!

SPA (Shortest Path Algorithms) project page

http://research.microsoft.com/en-us/projects/SPA/

Questions?

1. Introduction and Motivation
2. Definitions and Model
3. Classical Algorithms
4. Recent Algorithms
   - Reach
   - Contraction Hierarchies
   - Transit Nodes
5. Theoretical Results
   - Highway Dimension
   - Theoretical Bounds
6. Final Remarks