Resolution and SAT-solving

Topics today: ① Inference rules ② Resolution rule
③ Resolution algorithm ④ SAT-solvers

① Inference rules

A knowledge base (KB) is a set of sentences that are known to be true. (We can combine KB into a single sentence using $\wedge$).

e.g. $KB = \{ P, \; P \Rightarrow Q, \; \neg R, \; S \wedge T \}$

[same as $KB = \{ P \wedge P \Rightarrow Q \wedge \neg R \wedge S \wedge T \}$]

Can add to KB using inference rules:

e.g. "and elimination": if $\alpha \wedge \beta \in KB$, can add α to KB

Notation: $\boxed{\dfrac{\alpha \wedge \beta}{\alpha}}$

exercise: apply to above KB

"modus ponens": if $\alpha \in KB$ and $\alpha \Rightarrow \beta \in KB$, can add β to KB

Notation: $\boxed{\dfrac{\alpha, \; \alpha \Rightarrow \beta}{\beta}}$

exercise: apply to above KB

"resolution": see next section

② The Resolution inference rule

Resolution is an important inference rule.

Basic idea is that opposite literals in separate clauses cancel each other out, yielding a combined clause.

cancel

e.g. $P \vee Q \;,\; R \vee \neg Q$ yields $P \vee R$

combine

cancel

$P \vee Q \vee \neg R \vee S \;,\; Q \vee R \vee T \vee \neg U$

combine

yields $P \vee Q \vee S \vee T \vee \neg U$

Exercise: Apply resolution to the KB $\{ P \vee \neg Q,\ \neg P \vee R \vee \neg S,\ S \vee T \}$

The resolution rule is important because it can be used as part of an algorithm that infers entailment.
i.e. it decides whether $KB \models \alpha$ for any KB, α.

We study this next.

③ Our simple resolution algorithm for entailment

We want to determine whether $KB \models \alpha$.
Equivalently, is $KB \Rightarrow \alpha$ valid?
Equivalently, is $KB \wedge \neg\alpha$ unsatisfiable?

Algorithm:
- Convert $KB \wedge \neg\alpha$ to CNF
- Apply resolution repeatedly
- If you ever get an empty clause, conclude that $KB \models \alpha$
- If can't make any more clauses, conclude that $KB \not\models \alpha$.

This alg is guaranteed to terminate. The book has a proof, but we ~~don't~~ study it

Why? Because you've derived the empty clause, equiv to 'False', meaning $KB \wedge \neg\alpha$ is unsatisfiable

Why? Because you can now satisfy $KB \wedge \neg\alpha$. Detailed proof in book (not required) but basically just fill in the values

Exercise:

$$KB = \{ P \Rightarrow Q,\ Q \vee R \vee S,\ S \Rightarrow P \vee Q \}$$

(i) Does KB entail $Q \vee R$?
(ii) Does KB entail $\neg Q \wedge S$?

④ Efficiency of SAT-solvers

- Note that the resolution algorithm above is just a particular method of determining satisfiability, also known as <u>SAT-solving</u>.

- Satisfiability (or just "SAT") is of central importance in the theory of algorithms. It was the first problem to be proved <u>NP-complete</u>, in the early 1970s.

  → means:
  - no efficient algorithm is known to solve all instances

    efficient: i.e. polynomial time
  - if we did find an efficient alg, that alg would solve most other "hard" problems in CS
  - therefore, efficient alg for all instances probably doesn't exist (worst case)

- So, our resolution algorithm takes exponential time (in the number of variables and/or clauses)

- Better algorithms are known (e.g. Davis-Putnam), but still exponential in the worst case

- Still, "modern solvers handle problems with tens of millions of variables" – see last paragraph of book 7.6.1, p262

- An interesting special case where a linear time solution exists: if the KB consists completely of Horn clauses

clause with at most one positive literal

e.g. $\neg B \vee \neg C \vee D$, $\neg P \vee \neg Q$