# COMP 314 Homework Assignment 8

1. Explain why each of the following problems is in PolyCheck. Be sure to indicate which, if any, of the caveats in the definition of PolyCheck are used.

   (a) (5 points) The "threshold" version of the traveling salesperson problem. That is, the input consists of (i) a weighted undirected graph $G$, representing the distances between a set of cities; and (ii) a threshold value $T$. The output for a positive instance consists of an ordered list of the cities, representing a path in $G$ that visits each city (except the first) exactly once, starts and ends in the same place, and whose total length is at most $T$. If no such path exists, the problem is a negative instance and the output is "no".

   (b) (5 points) Let's call this problem INTEGERDIVISION. The input consists of two integers, $M$ and $N$, written in decimal notation and separated by a space character. If $M$ is divisible by $N$, the output is $M/N$, again written in decimal notation. If $M$ is not divisible by $N$, the output is "no".

   (c) (5 points) Let's call this problem ANONYMOUSFACEBOOK. The input consists of two graphs, $L$ (for Labeled) and $A$ (for Anonymous). The graph $L$ consists of vertices labeled with peoples' names, and edges between any two people that are friends on Facebook. (We are working in the fictional universe here, so $L$ does not have to represent data from the real-life version of Facebook.) The graph $A$ has the same format, but the vertices are labeled with anonymous strings rather than peoples' names. The ANONYMOUSFACEBOOK problem asks: could $L$ and $A$ represent the same set of people? (Note: ANONYMOUSFACEBOOK is just a disguised version of a famous problem called GRAPHISOMORPHISM, which we will probably discuss in class at some point.)

2. (5 points) Consider the following decision problem, called IMPOSSIBLETOBALANCE. The input is a list of integers, such as "45 23 4 3 72 12". The output is "yes", if it is impossible to partition the integers into two sets that *balance*—that is, the sum of each set is the same—and otherwise, the output is "no". For example: on input "1 2 4" the output is "yes" (because it's impossible to partition the input into two balanced sets); and on input "1 2 4 1 2" the output is "no" (because we can balance $1 + 2 + 2 = 4 + 1$). Your friend claims that IMPOSSIBLETOBALANCE is in PolyCheck, because we can provide as a hint a list of all partitions of the integer inputs, together with the weights of these partitions. Explain why your friend's reasoning is not correct.

3. (5 points) Consider again the ANONYMOUSFACEBOOK problem defined above. By directly using the definition of NPoly, prove that ANONYMOUSFACEBOOK is in NPoly.

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4. (5 points) Let EXACTPACKING be the same as the PACKING problem defined in class, except that instead of allowing the total weight of the selected packages to range between $L$ and $H$, the total weight must exactly equal $H$. That is, the number $L$ is no longer part of the input and we insist that the truck is packed exactly with its maximum load $H$. Give a polynomial time reduction from EXACTPACKING to PACKING.

5. (5 points) Define the STRANDEDSALESPERSON problem to be exactly the same as TRAVELINGSALESPERSON, except that the salesperson does not have to start and end in the same city. (And assume we are using the "threshold" versions for both of these problems.) Give a polynomial time reduction from STRANDEDSALESPERSON to TRAVELINGSALESPERSON.