COMP 314 Homework Assignment D

This is version 3 of the assignment, published 2/13/15. Questions D3 and D6 have changed somewhat.

Chapter 7

Question D1. (10 points) The program `isOddViaReduction.py` in Figure 7.2 reduces IsOdd to LastDigitIsEven. Write a similar program that does the reverse, reducing LastDigitIsEven to IsOdd.

Question D2. (15 points) Consider the decision problem IntOnString, which takes two strings P, I as input and outputs "`yes`" if $P(I)$ is a nonnegative integer (i.e. a string containing only digits) and "`no`" otherwise. Following the pattern of Figure 7.4, write two Python programs that together demonstrate a reduction from YesOnString to IntOnString.

Question D3. (Ungraded) Consider the decision problem IntOnAllInts. This problem takes a single string P as input, and outputs "`yes`" if $P(I)$ is a nonnegative integer for all input strings I which are themselves nonnegative integers. In other words, it answers the question: "Does P output a nonnegative integer for all nonnegative integer inputs?" Prove that IntOnAllInts is undecidable. You may do so explicitly using Python programs, or use prose descriptions of the programs involved.

Question D4. (3 points) Suppose $F_1, F_2, \ldots, F_{10}$ are computational problems such that each problem reduces to the next one: F_1 reduces to F_2, F_2 reduces to F_3, and so on. Suppose also that F_3 is computable and F_5 is uncomputable. Which of the other problems are therefore computable? Which are uncomputable? Which could be either computable or uncomputable?

Question D5. (3 points) Suppose F and G are general computational problems, and D is a decision problem. Also suppose F reduces to G and D reduces to G. What, if anything, can we conclude if:

(a) D is undecidable?

(b) G is computable?

(c) F is computable?

Question D6. (15 points) Prove that EmptyOnEmpty is undecidable. That is, given a program P, the question of whether $P(\lambda) = \lambda$ is undecidable. You are not required to give explicit Python programs. Explain the necessary reduction(s) in prose.

Question D7. (Ungraded) Repeat the previous question for another undecidable problem of your choice, such as KoalaOnEmpty, NoOnSome, or YesOnKoala.

Question D8. (Ungraded) In mathematics, a *fixed point* of a function f is a value x such that $f(x) = x$. For a program P, we can similarly define a *fixed string* as a string I such that $P(I) = I$. We can then define the decision problem HasFixedString which takes an input P and outputs "`yes`" if and only if P has a fixed string. Prove that HasFixedString is undecidable. Your proof may be in the practical style of this book (using mostly Python programs, and a brief explanation of why the programs prove the claim) or in a more standard abstract mathematical style.

Question D9. (Ungraded) Define the computational problem NumLonger as follows. The input is a program P. The solution is the number of distinct strings I for which $P(I)$ is defined and is longer than I. In other words, the solution is the cardinality of the following set:

$$\{I \in \text{ASCII}^* \text{ such that } |P(I)| > |I|\}$$

Of course, this set could be infinite, in which case the solution is the string "`infinite`". Is NumLonger computable? Give a proof of your answer.

Question D10. (Ungraded) Our proof that all four variants of the halting problem are undecidable (page 146) omitted some details. Fill in one of these missing details as follows: write a Python program demonstrating a reduction from HaltsOnString to HaltsOnSome.

Question D11. (10 points) Consider our proof that NumCharsOnString is uncomputable. In particular, examine the program `alterYesToNumChars.py` (top of Figure 7.12). Line 9 of this program returns the string "`xx`", but the proof could remain valid even if we had used a different return value here. Which return values can be used at line 9? In particular, for which of the following return values would the proof remain valid: λ, "`a`", "`aa`", "`aaa`", "`aaaa`"? Explain your answer.

Question D12. (Ungraded) Define the decision problem SlowerThanInputLength as follows. The input is a program P. The solution is "`yes`" if and only if the number of steps before terminating on input I is more than $|I|$, for all I. In other words, SlowerThanInputLength answers the question: "is the running time of P always more than the length of its input?" (Note we define the running time as infinite whenever $P(I)$ is undefined.) Prove that SlowerThanInputLength is undecidable.

Question D13. (10 points) Let $P =$"`def P(x):  return str(5*len(x))`".

(a) Give an example of a computational problem F such that P is a positive instance of $\textsc{Computes}_F$.

(b) Give an example of a computational problem G such that P is a negative instance of $\textsc{Computes}_G$.

Question D14. (10 points) Use the Uncomputability of $\textsc{Computes}_F$ (or one of its variants, such as Rice's theorem) to prove that each of the following problems is uncomputable:

(a) $\textsc{ComputesLength}$: input is program P, solution is "`yes`" if and only if $P(I) = |I|$ for all I.

(b) $\textsc{SearchesForSubstring}$: input is program P, solution is "`yes`" if and only if P searches for some fixed substring. That is, there exists a string s such that $P(I) =$ "`yes`" if and only if s is a substring of I.

Question D15. (15 points) State whether each of the following instances of the Post correspondence problem is positive or negative. Explain your answers.

(a) $\left\{ \begin{array}{|c|}\hline \texttt{C} \\ \hline \texttt{A} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{G} \\ \hline \texttt{T} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{A} \\ \hline \texttt{C} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{T} \\ \hline \texttt{G} \\ \hline \end{array} \right\}$

(b) $\left\{ \begin{array}{|c|}\hline \texttt{C} \\ \hline \texttt{CC} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{A} \\ \hline \texttt{AA} \\ \hline \end{array} \right\}$

(c) $\left\{ \begin{array}{|c|}\hline \texttt{G} \\ \hline \texttt{CG} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{C} \\ \hline \texttt{CG} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{CG} \\ \hline \texttt{T} \\ \hline \end{array}, \begin{array}{|c|}\hline \texttt{GAT} \\ \hline \texttt{A} \\ \hline \end{array} \right\}$

Total points on this assignment: 91