

COMP 314 Homework Assignment G

Chapter 12

Question G1. Explain why each of the following problems is in PolyCheck. Be sure to indicate which, if any, of the caveats in the caveat-based definition of PolyCheck are used.

- (a) (10 points) The “decision” version of the undirected Hamiltonian cycle problem. That is, the input consists of a weighted undirected graph G . The solution is “yes” if G has a Hamiltonian cycle, and “no” otherwise.
- (b) (10 points) Let’s call this problem INTEGERDIVISION. The input consists of two integers, M and N , written in decimal notation and separated by a space character. If M is divisible by N , the solution is M/N , again written in decimal notation. If M is not divisible by N , the output is “no”.
- (c) (10 points) Let’s call this problem ANONYMOUSFACEBOOK. The input consists of two graphs, L (for Labeled) and A (for Anonymous). The graph L consists of vertices labeled with peoples’ names, and edges between any two people that are friends on Facebook. (We are working in a fictional universe here, so L does not have to represent data from the real-life version of Facebook.) The graph A has the same format, but the vertices are labeled with anonymous strings rather than peoples’ names. The ANONYMOUSFACEBOOK problem asks: could L and A represent the same set of people? (Note: ANONYMOUSFACEBOOK is just a disguised version of a famous problem called GRAPHISOMORPHISM, which we will probably discuss in class at some point.)

Question G2. (10 points) Consider the following decision problem, called IMPOSSIBLETOBALANCE. The input is a list of integers, such as “45 23 4 3 72 12”. The solution is “yes” if it is impossible to partition the integers into two sets that *balance*—that is, the sum of each set is the same. Otherwise, the output is “no”. For example: on input “1 2 4” the solution is “yes” (because it’s impossible to partition the input into two balanced sets); and on input “1 2 4 1 2” the solution is “no” (because we can balance $1 + 2 + 2 = 4 + 1$). Your friend claims that IMPOSSIBLETOBALANCE is in PolyCheck, because we can provide as a hint a list of all partitions of the integer inputs, together with the weights of these partitions. Explain why your friend’s reasoning is not correct.

Question G3. (10 points) Consider again the ANONYMOUSFACEBOOK problem defined above. By directly using the definition of NPoly, prove that ANONYMOUSFACEBOOK is in NPoly.

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The next two questions ask for polynomial time reductions. You may explain the reductions in English. There is no need to write Python code. However, the explanations should be clear enough that they could be used to write the corresponding Python code.

Question G4. (10 points) Let EXACTPACKING be the same as the PACKING problem defined in the textbook, except that instead of allowing the total weight of the selected packages to range between L and H , the total weight must exactly equal H . That is, the number L is no longer part of the input and we insist that the truck is packed exactly with its maximum load H . Give a polynomial time reduction from EXACTPACKING to PACKING.

Question G5. (20 points) Define the “stranded salesperson problem”, SSP, to be exactly the same as TSP, except that the salesperson does not have to start and end in the same city. In other words, SSP asks for the shortest Hamiltonian path rather than Hamiltonian cycle. Let the decision version of SSP be SSPD. SSPD is defined in a similar way to TSPD in the textbook: it takes an additional threshold L as input, and the solution is “yes” if there is a Hamiltonian path of length at most L . Give a polynomial time reduction from SSPD to TSPD.

Total points on this assignment: 80