Claim HoS is undecidable

Assume $H(P,I)$ returns $\begin{cases} \text{'yes' if } P \text{ halts on } I \\ \text{'no' otherwise} \end{cases}$

Consider $W(P)$:

1. $v = H(P,P)$
2. if $v =$ 'yes', loop
   else halt

Does W halt on W? 2 cases:

a) W halts on W i.e. $H(W,W) =$ 'yes'.

But this means $W(W)$ loops. #

b) W loops on W i.e. $H(W,W) =$ 'no'.

But this means $W(W)$ halts. #

Claim $HET \notin$ Poly

Assume • $H(P,I)$ returns $\begin{cases} \text{'yes' if } P \text{ halts in } < 2^{|I|} \text{ steps} \\ \text{'no' otherwise.} \end{cases}$

and • $H(P,I)$ requires $\leq r(|P|+|I|)$, some poly r.

Consider $W(P)$:

1. $v = H(P,P)$
2. if $v =$ 'yes', loop
   else halt

Does $W(W)$ halt in $< 2^{|W|}$ steps? 2 cases:

a) $W(W)$ halts in $< 2^{|W|}$ steps i.e. $H(W,W) =$ 'yes'.

But this means $W(W)$ loops. #

b) $W(W)$ halts in $\geq 2^{|W|}$ steps i.e. $H(W,W) =$ 'no'.

Then running time of W is $\leq$

$$\underset{\text{some const}}{C} + \underset{\text{bound for } H}{r}(2|W|)$$

Thus $\underbrace{C + r(2|W|)}_{\text{poly}} \geq \underbrace{2^{|W|}}_{\text{exp}}$

Contradiction for sufficiently large $|W|$ #.