Algorithms do change the world

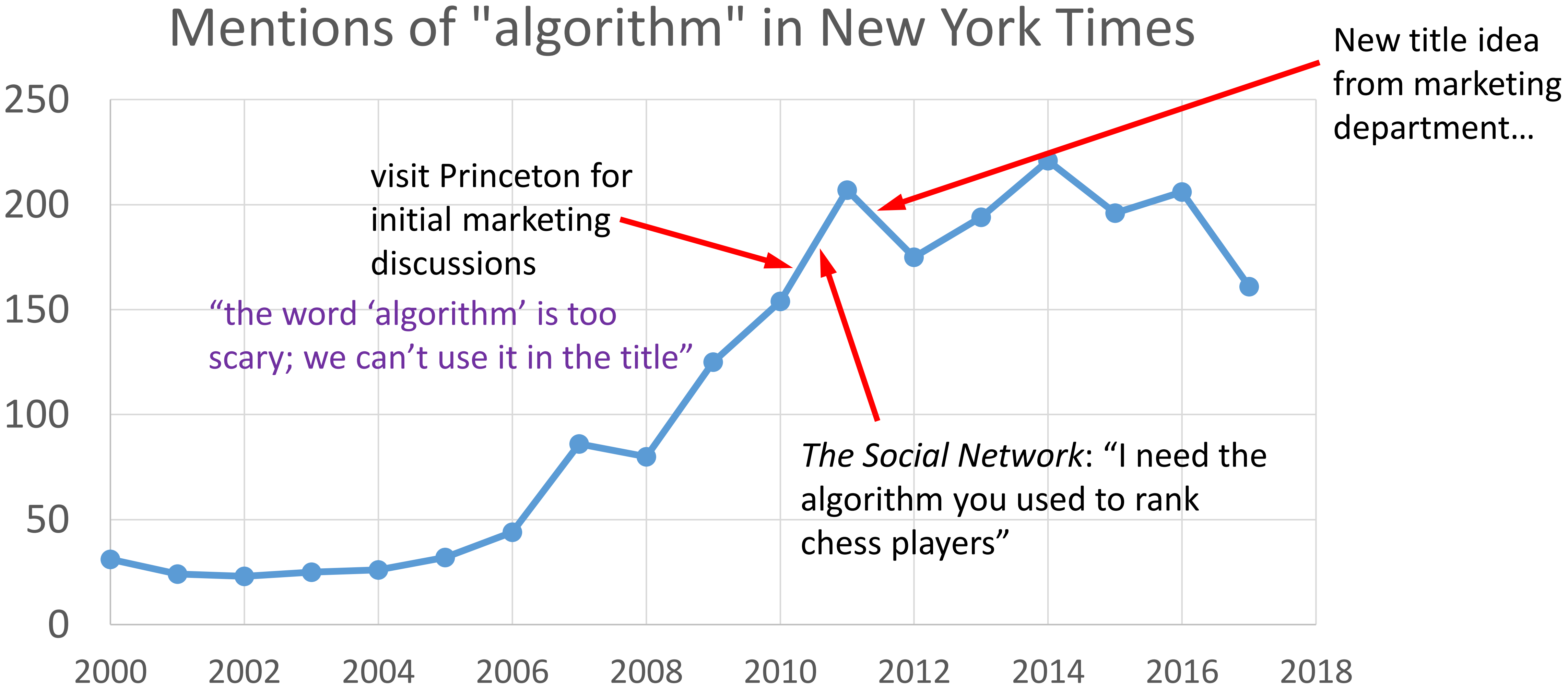
John MacCormick

Dickinson College

Outline: life and learning in the age of algorithms

- Are we living in an age of algorithms?
- What is an algorithm?
- What algorithmic ideas should be in the school-age maths curriculum?

Are we living in an age of algorithms?



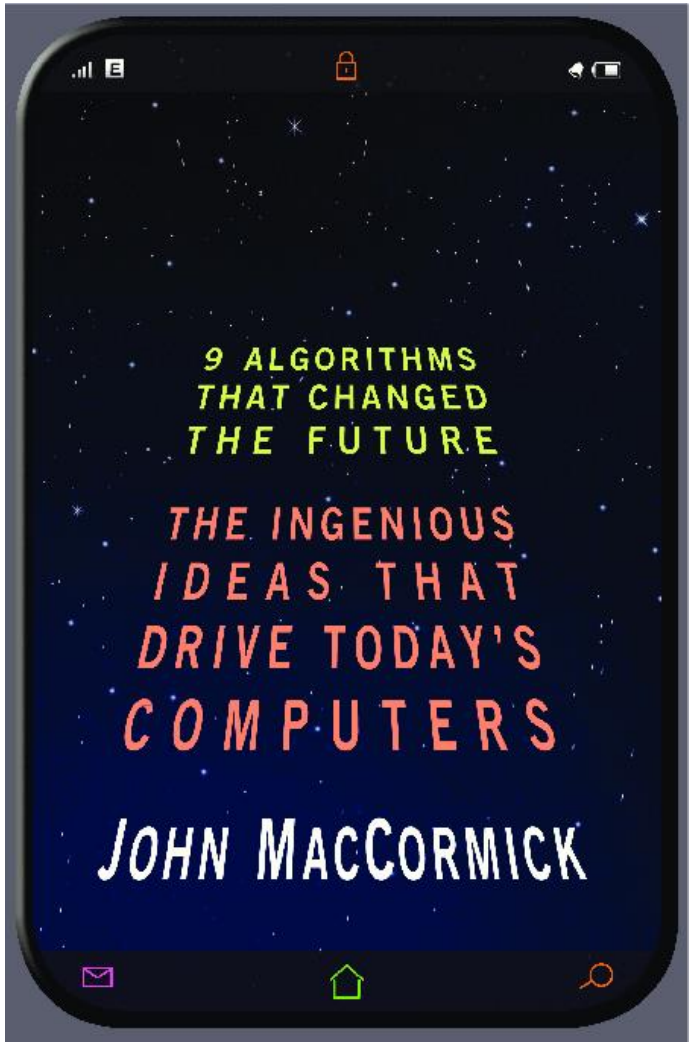
visit Princeton for initial marketing discussions

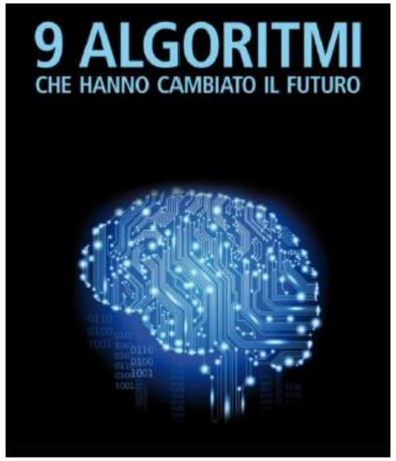
“the word ‘algorithm’ is too scary; we can’t use it in the title”

The Social Network: “I need the algorithm you used to rank chess players”

New title idea from marketing department…

… "9 Algorithms That Changed the Future"!


9 ALGORITHMS THAT CHANGED THE FUTURE
THE INGENIOUS IDEAS THAT DRIVE TODAY'S COMPUTERS
JOHN MACCORMICK



acornLoft
컴퓨터 세상을 만든 기발한 아이디어들
미래를 바꾼 아홉 가지 알고리즘
Nine Algorithms That Changed the Future
The Ingenious Ideas That Drive Today's Computers

世界でもっとも強力な
9のアルゴリズム
Nine Algorithms that Changed the Future
The Ingenious Ideas that Drive Today's Computers
ジョン・マコーミック
John MacCormick
わたしたちは
アルゴリズムの中で
日常暮らしている
ネット時代のアルゴリズムを直観で理解する
日経BP社

What is an algorithm?

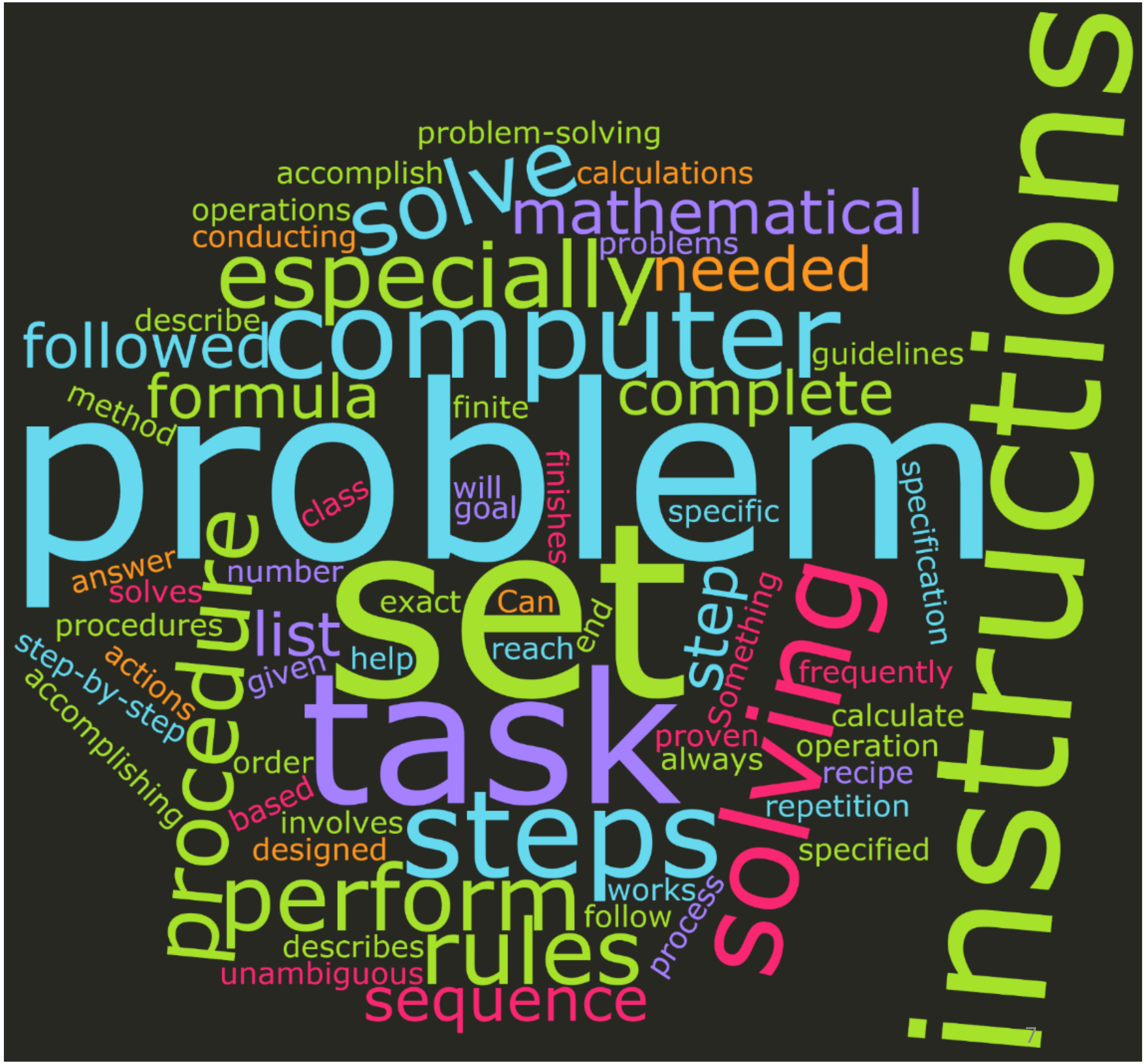
“at first glance it may look as though someone intended to write ‘logarithm’ but jumbled up the first four letters”

Donald E. Knuth (1968), *The Art of Computer Programming*

What is an algorithm?

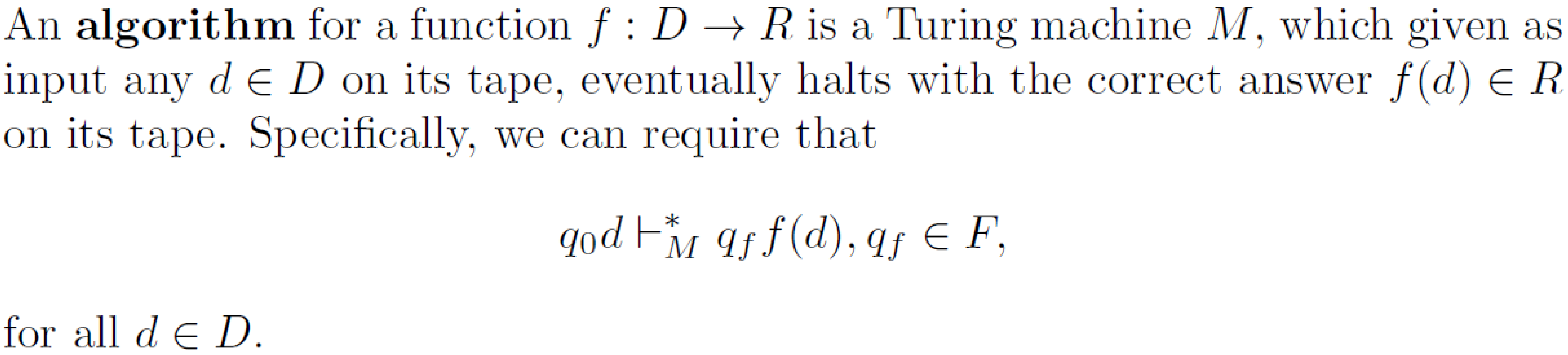
- Please take 30 seconds to come up with your own definition

What is an algorithm?



Source: top 15 definitions of “algorithm” from Google, processed by wordclouds.com

What is an algorithm?



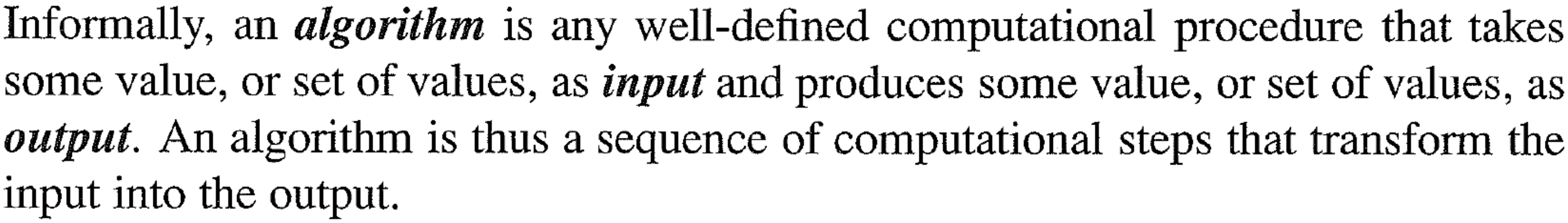
An **algorithm** for a function $f : D \to R$ is a Turing machine M, which given as input any $d \in D$ on its tape, eventually halts with the correct answer $f(d) \in R$ on its tape. Specifically, we can require that

$$q_0 d \vdash_M^* q_f f(d), q_f \in F,$$

for all $d \in D$.

Linz, *An Introduction to Formal Languages and Automata*

What is an algorithm?



Informally, an ***algorithm*** is any well-defined computational procedure that takes some value, or set of values, as ***input*** and produces some value, or set of values, as ***output***. An algorithm is thus a sequence of computational steps that transform the input into the output.

Cormen-Leiserson-Rivest-Stein, *Introduction to Algorithms*

Examples of famous algorithms

- Please take 30 seconds to think of one or two algorithms that you have heard of

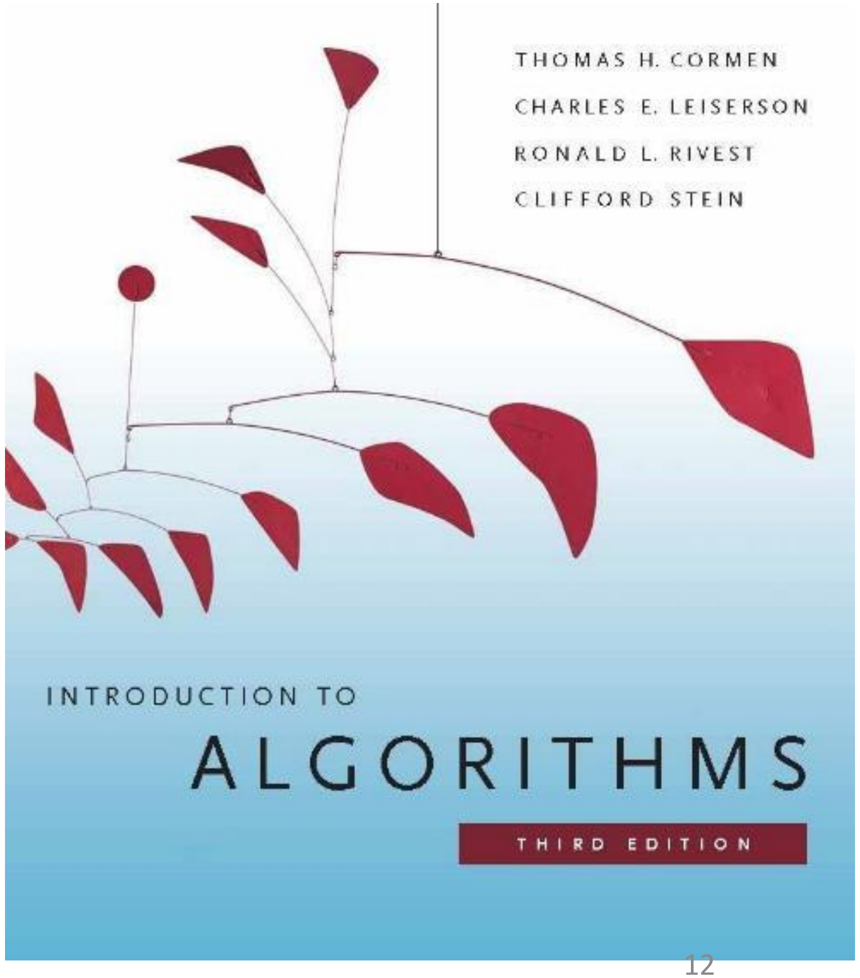
Examples of famous algorithms

- Quicksort
- PageRank
- Fast Fourier transform
- Euclid’s algorithm (GCD)
- Dijkstra’s algorithm (shortest path in a graph)

Next: “nuts-and-bolts” algorithms versus “niche” algorithms

“Nuts-and-bolts” algorithms are used as building blocks in most computer programs

- Examples: sorting algorithms, hash tables, …



“Niche” algorithms solve more specific problems

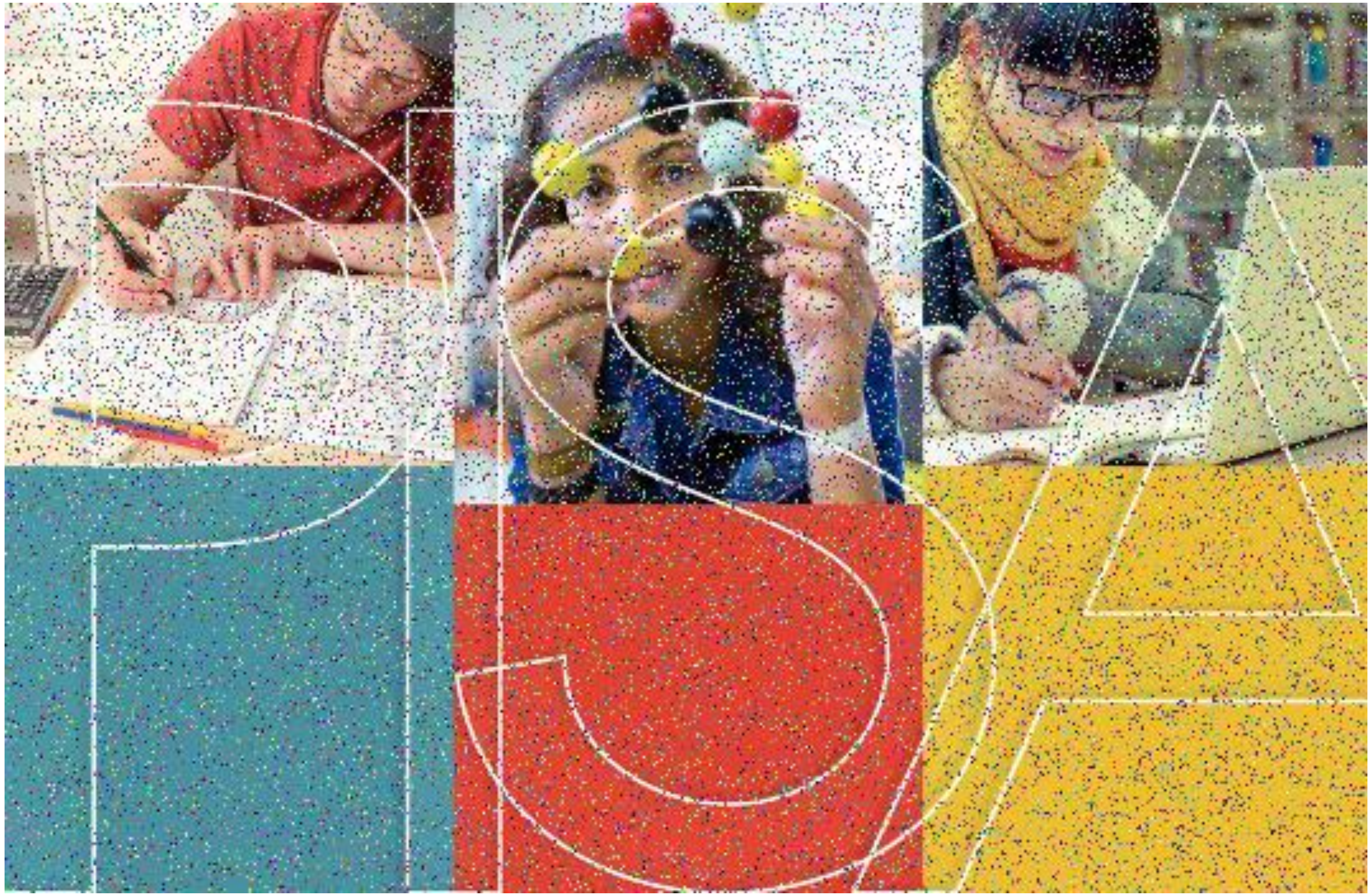
- A personal selection:
  - Decision trees
  - Error correcting codes
  - Compression
  - Digital signatures

Next: examine two specific algorithms to understand more details:

1. 2D parity error correcting code
2. Grade school multiplication

ALGORITHMS TO LIVE BY
THE COMPUTER SCIENCE OF HUMAN DECISIONS
BRIAN CHRISTIAN & TOM GRIFFITHS
‘Practical and highly enjoyable’ POPULAR SCIENCE
GORITHMS CHANGED FUTURE
GENIOUS THAT ODAY'S TERS
ORMICK

Without error correction, your upload to Facebook would look like this



PISA 2015 Results (Volume I), http://dx.doi.org/10.1787/19963777

One solution: 2D parity error correcting code

4 8 3 7 2 5 4 3 6 8 2 7 5 6 5 3 9 9 7 8 4 3 0 6

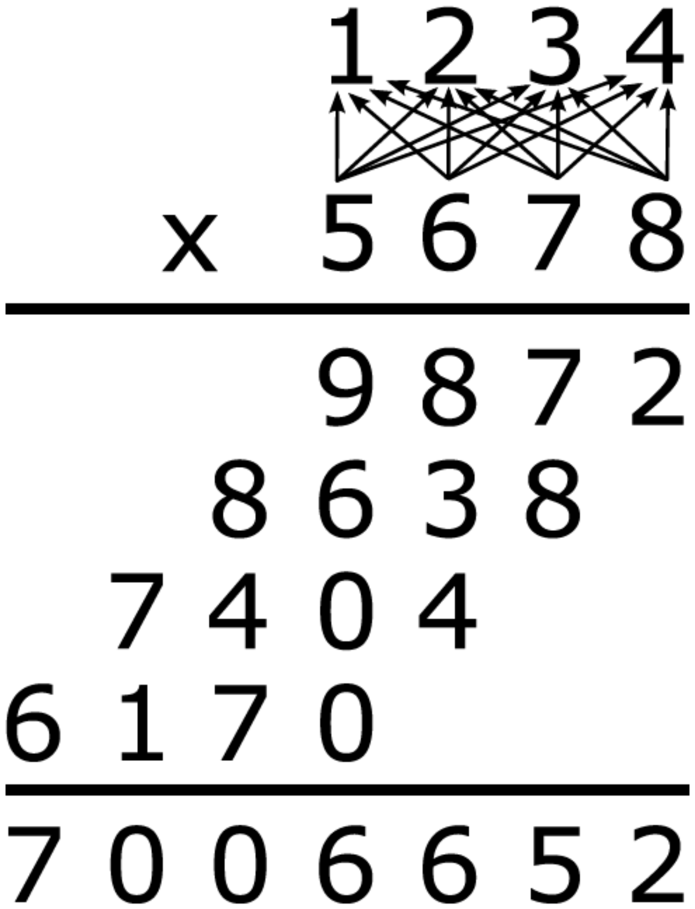
4	8	3	7	2
5	4	3	6	8
2	7	5	6	5
3	9	9	7	8
4	3	0	6	

4	8	3	7	2	2
5	4	3	6	8	8
2	7	5	6	5	0
3	9	9	7	8	8
4	3	0	6		
4	8	0	6		

This “2D parity” algorithm has many connections to mathematics (and most other algorithms have similar mathematical relevance)

- Modular arithmetic: checksum is computed modulo 10
- Algebra: formula for corrected digit
- Proof: Is it guaranteed to correct any single error? Detect multiple errors?
- Probability and statistics: what is the chance of an undetected error?

The “grade school multiplication” algorithm provides another useful example

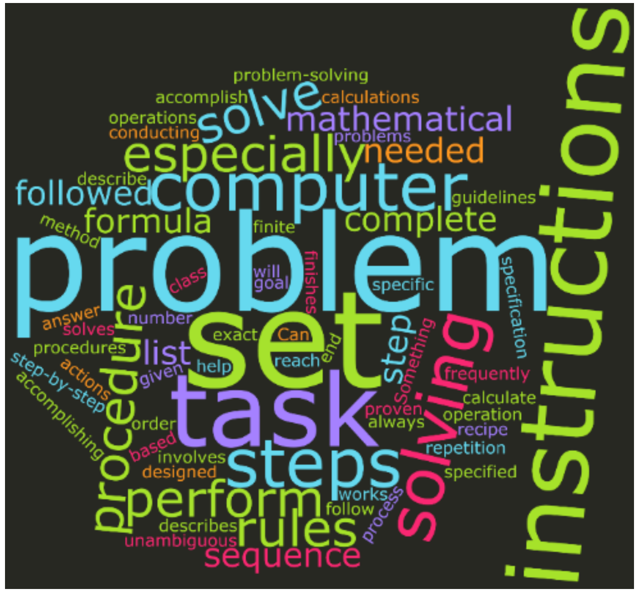


```
      1 2 3 4
  x   5 6 7 8
-------------
      9 8 7 2
    8 6 3 8
  7 4 0 4
6 1 7 0
-------------
7 0 0 6 6 5 2
```

- If we double the number of digits, how much longer does it take?

- This is an example of *complexity theory*

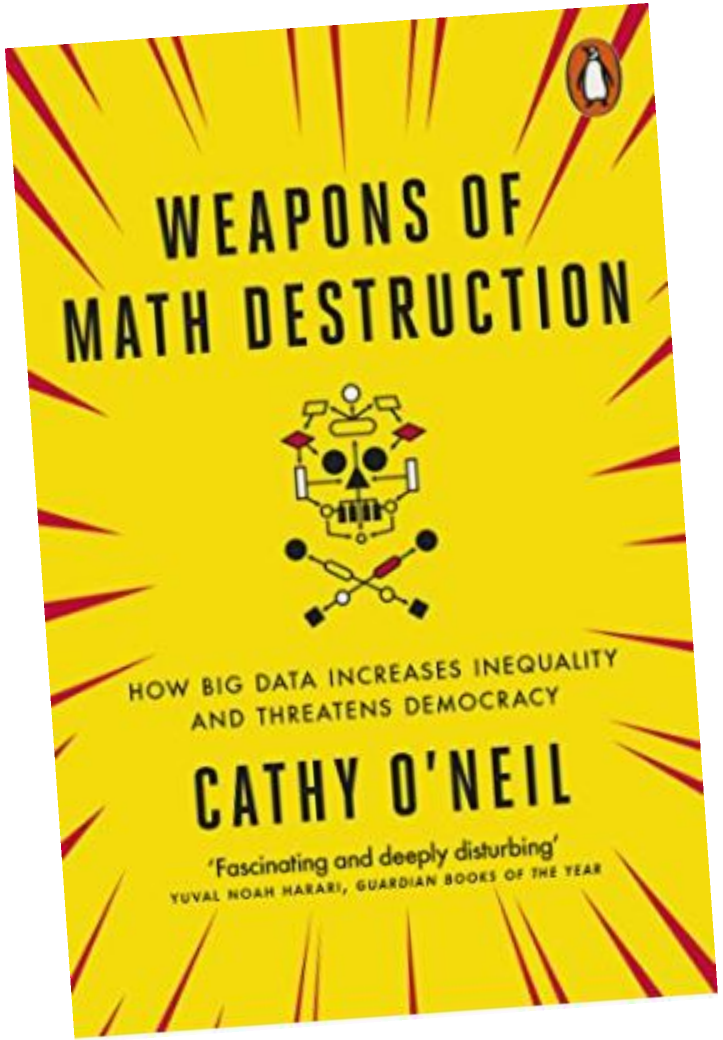
Summary: what is an algorithm?

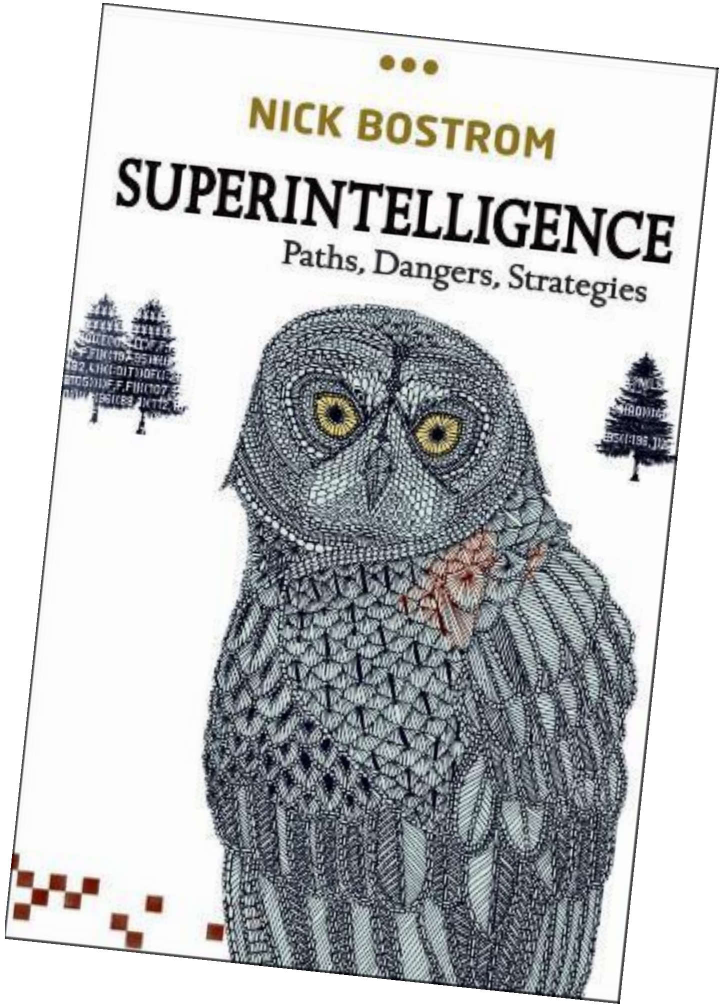


$$q_0 d \vdash_M^* q_f f(d), q_f \in F,$$

- Method for solving a problem using a computer program
- Usually based on mathematical techniques

Why should citizens know about algorithms?


WEAPONS OF
MATH DESTRUCTION
HOW BIG DATA INCREASES INEQUALITY
AND THREATENS DEMOCRACY
CATHY O'NEIL
'Fascinating and deeply disturbing'
YUVAL NOAH HARARI, GUARDIAN BOOKS OF THE YEAR



Outline: life and learning in the age of algorithms

- Are we living in an age of algorithms?
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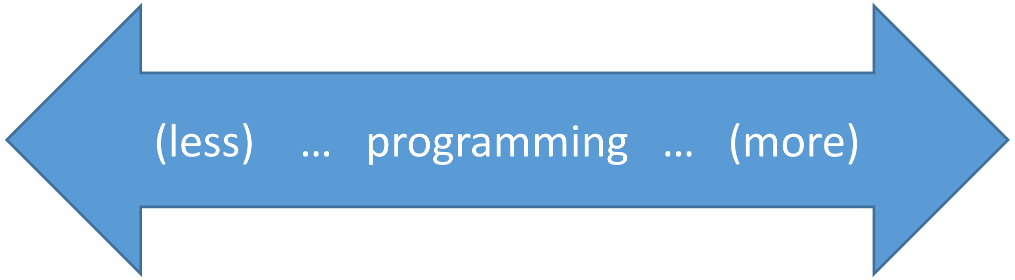
What algorithmic ideas should be in the school-age maths curriculum?

- Perhaps, none
  - i.e. algorithmic thinking is important, but not important enough to displace essential maths
- If we do want algorithms, a wide spectrum of approaches is possible
  - Two extremes on this spectrum:

Unplugged

Algorithms with no programming, no computers



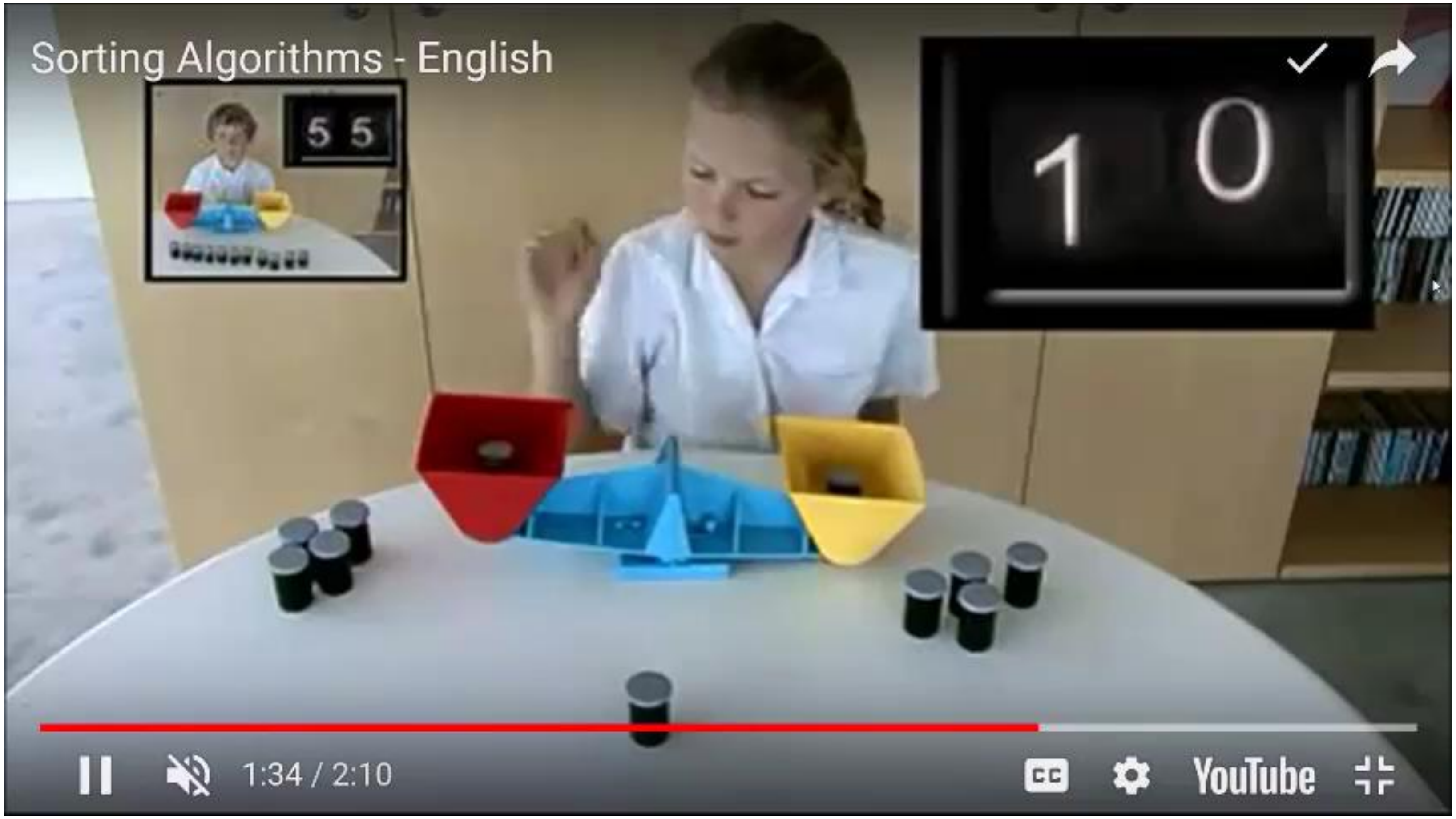
Integrated

Algorithms fully integrated with programming and maths

Example: CSunplugged.org

Example: bootstrapworld.org

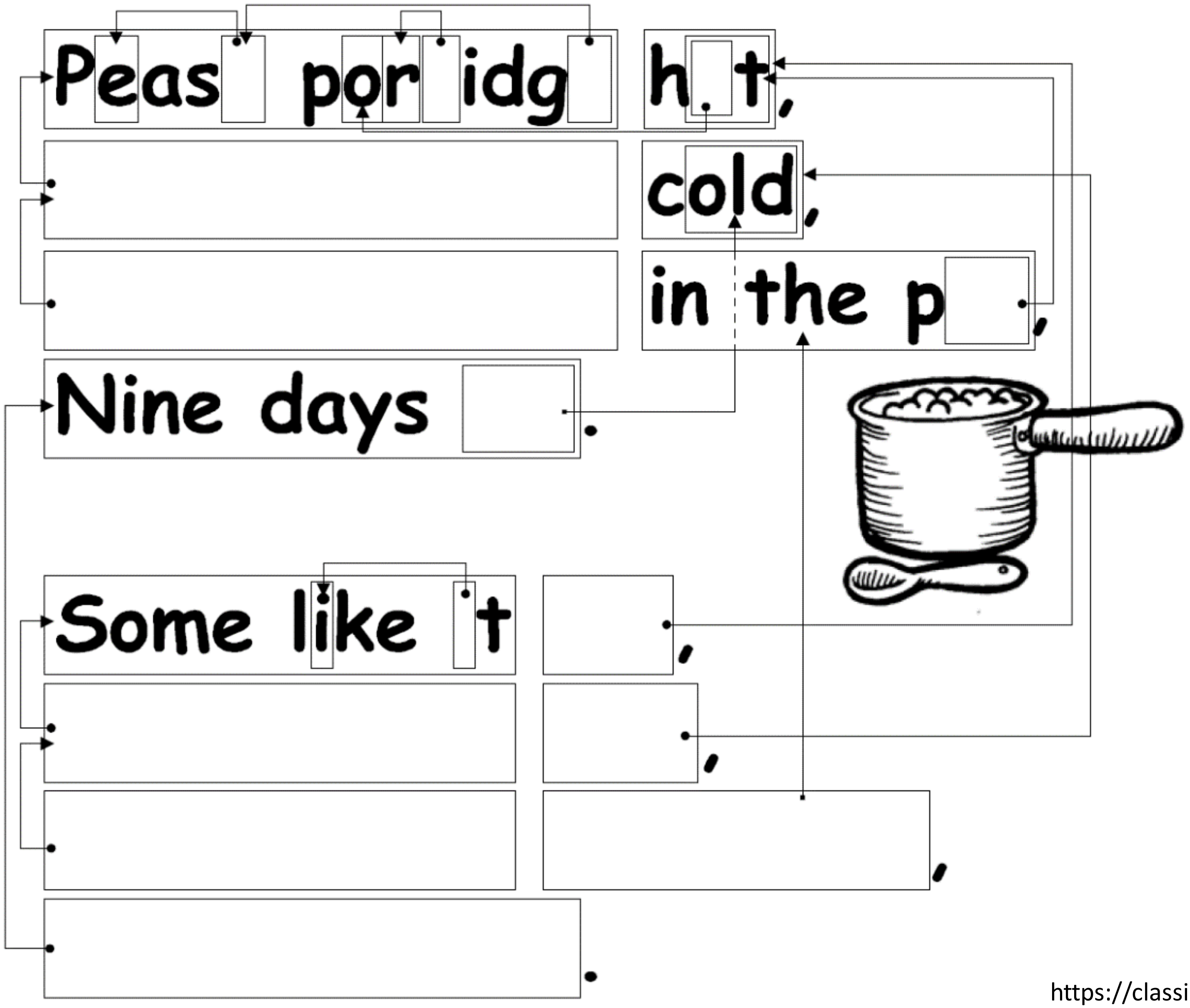
Teaching algorithmic thinking with no computers and no programming (CSunplugged.org)



Yes… this is QuickSort!

https://classic.csunplugged.org/sorting-algorithms/

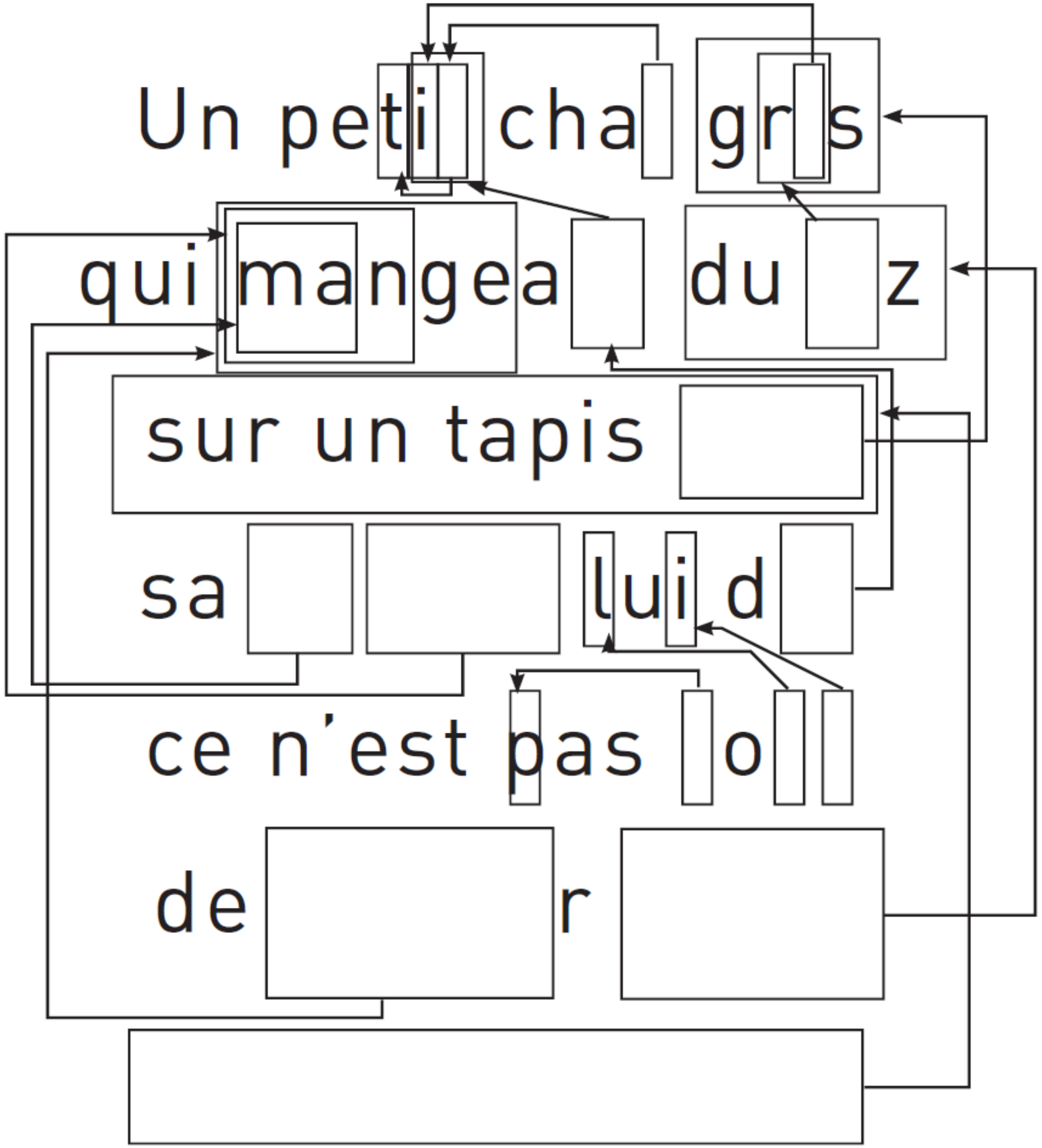
A classic compression algorithm from CSunplugged.org



Yes… this is LZ compression, as used in ZIP files!

https://classic.csunplugged.org/text-compression/

A classic compression algorithm from CSunplugged.org

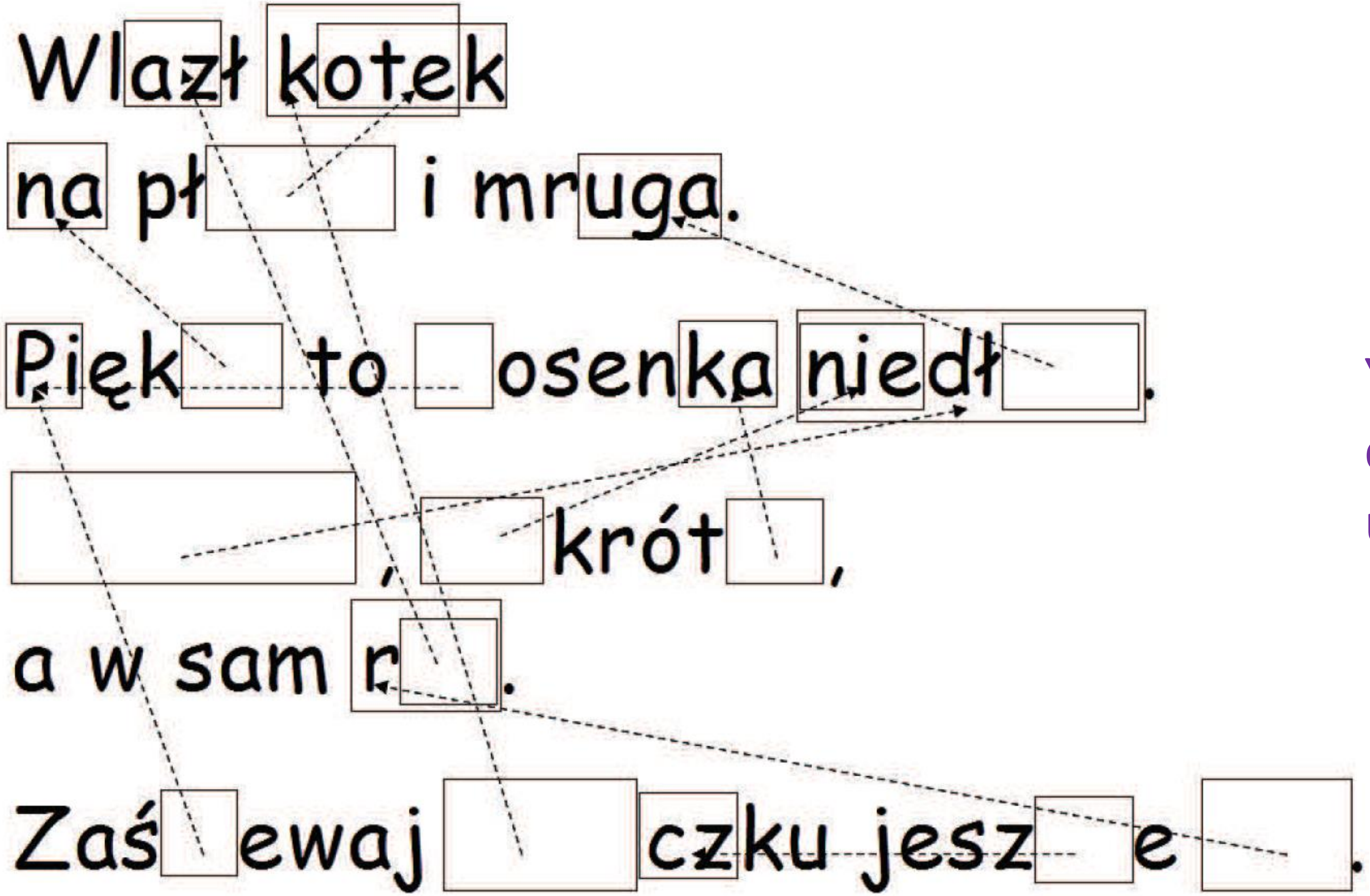


Un peti cha gr s
qui mangea du z
sur un tapis
sa lui d
ce n’est pas o
de r

Yes… this is LZ compression, as used in ZIP files!

https://classic.csunplugged.org/text-compression/

A classic compression algorithm from CSunplugged.org

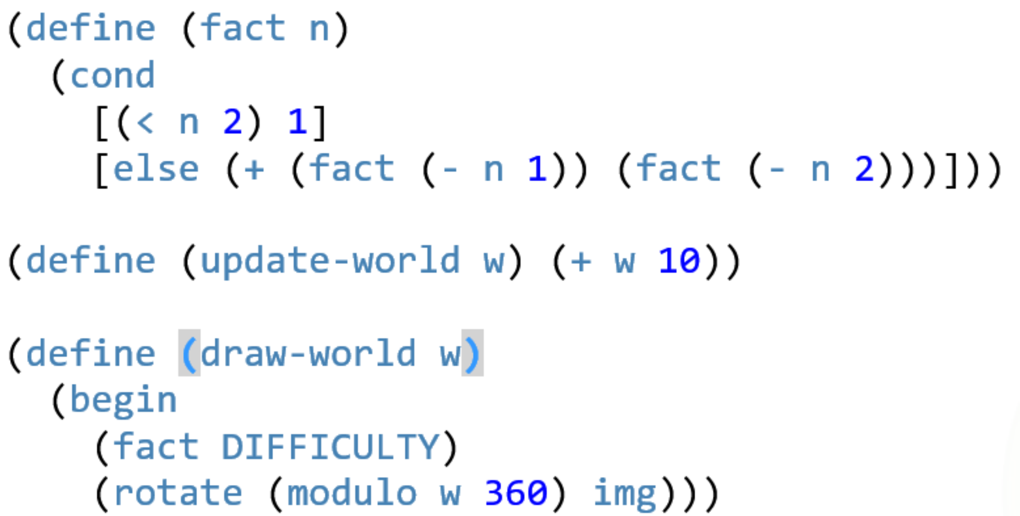


Yes… this is LZ compression, as used in ZIP files!

https://classic.csunplugged.org/text-compression/

Example: the bootstrapworld.org Algebra course

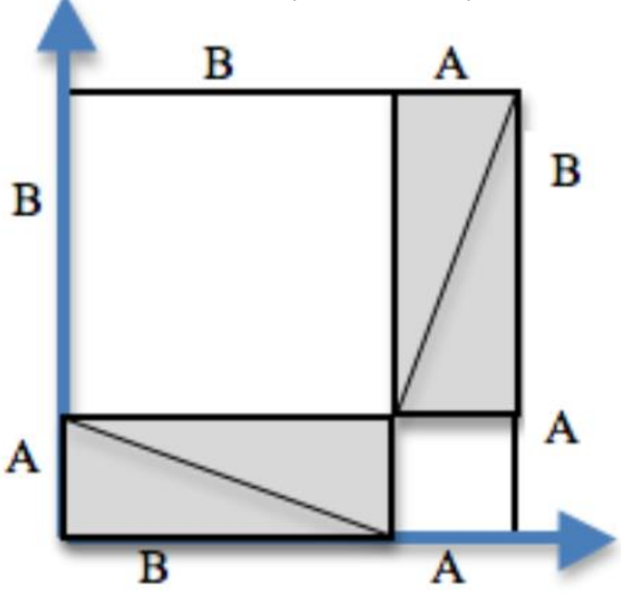
- Maths + programming:
  - Pencil-and-paper workbooks
  - Write code in browser
- Algebra course content:
  - Cartesian coordinates
  - Functions, domain, range
  - Derive, discuss, and prove the Pythagorean theorem
  - Then *use* the Pythagorean theorem to detect collisions in a video game



```
(define (fact n)
  (cond
    [(< n 2) 1]
    [else (+ (fact (- n 1)) (fact (- n 2)))]))

(define (update-world w) (+ w 10))

(define (draw-world w)
  (begin
    (fact DIFFICULTY)
    (rotate (modulo w 360) img)))
```

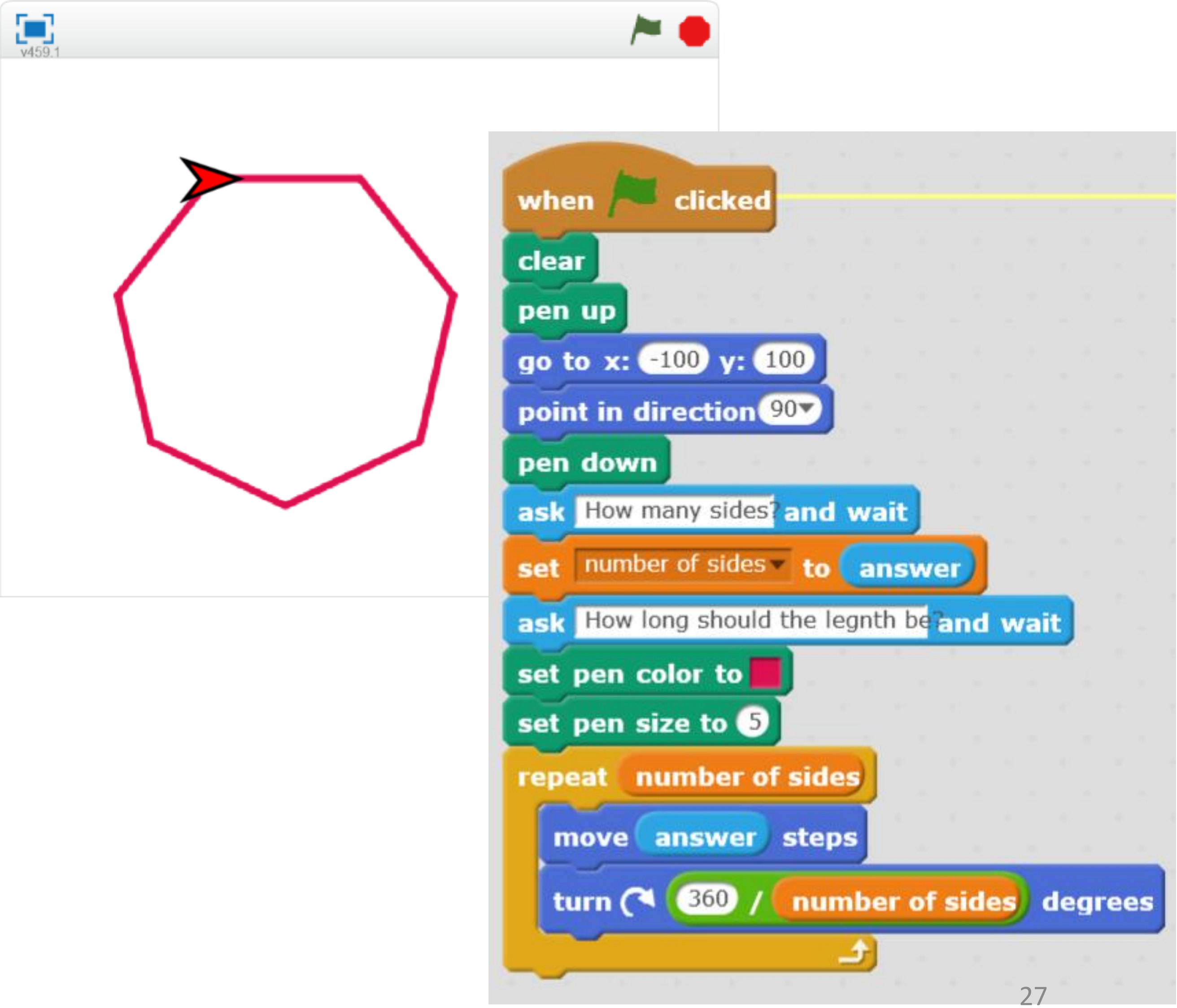


http://www.bootstrapworld.org/materials/algebra/

Other examples of integrating mathematics with programming

- Draw n-gon in Scratch

- Integration via trapezoidal approximation



```
when ⚑ clicked
clear
pen up
go to x: -100 y: 100
point in direction 90▾
pen down
ask How many sides? and wait
set number of sides▾ to answer
ask How long should the legnth be? and wait
set pen color to 🟥
set pen size to 5
repeat number of sides
    move answer steps
    turn ↻ 360 / number of sides degrees
```

https://scratch.mit.edu/projects/12600956

Both ends of the spectrum have advantages

Unplugged

(less) … programming … (more)

Integrated

Advantages:

- No setup time
- No software problems
- Low barrier for instructors

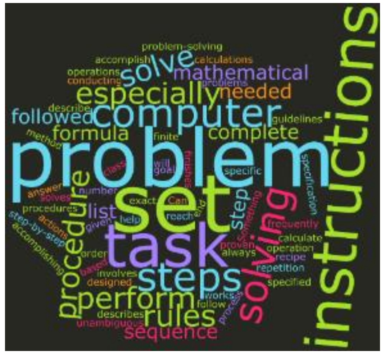
Advantages:

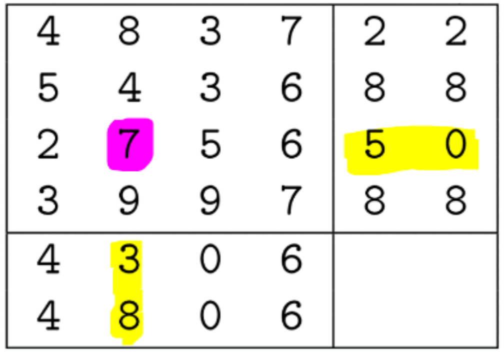
- STEM equity (all students exposed to code+maths)
- Potentially superior learning outcomes due to active learning

Summary: life and learning in the age of algorithms

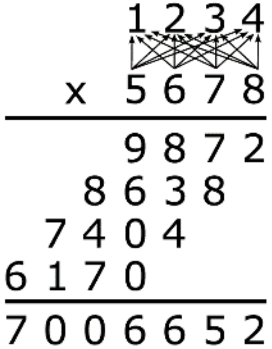
- Are we living in an age of algorithms?
  - Yes, but don’t confuse algorithms with technology

- What is an algorithm?





4	8	3	7	2	2
5	4	3	6	8	8
2	7	5	6	5	0
3	9	9	7	8	8
4	3	0	6		
4	8	0	6		



```
          1 2 3 4
      x   5 6 7 8
    -------------
          9 8 7 2
        8 6 3 8
      7 4 0 4
    6 1 7 0
    -------------
    7 0 0 6 6 5 2
```

- What algorithmic ideas should be in the school-age maths curriculum?
  - 3 possible approaches: None, Unplugged, Integrated