Dynamical Effects of Non-Linearities and Time-Varying Gain Modulation in Neurally Plausible Network Models of Perceptual Decision-Making

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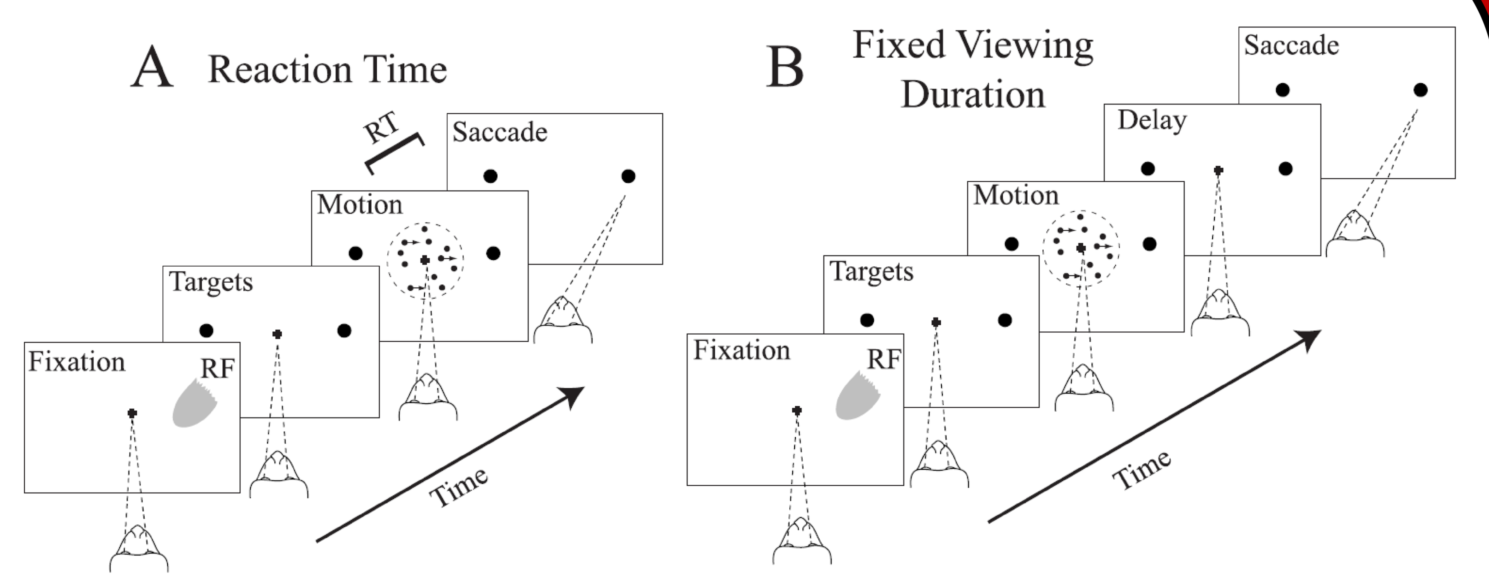
Overview

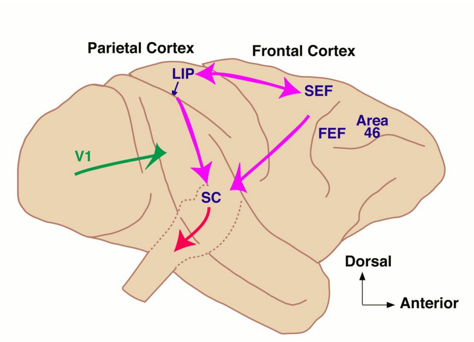
- **Simple perceptual decision-making links sensation to action, constitutes the basis of many cognitive processes.**
- **How does the underlying neuronal network operate, and how can it be adapted or modified to achieve behavioral goals ?**
- **Mathematical models of these decision processes provide a theoretical framework against which experimental data can be interpreted and evaluated.**
- **We employ computational and analytical techniques from Dynamical Systems theory to study low-dimensional neural network models of decision-making in Two-Alternative Forced-Choice tasks.**
- **How can non-linearities in and multiplicative gain modulation of neural input-output (transfer) functions (e.g. due to attentional processes) affect the dynamics of the decision network ?**

Two-Alternative Forced-Choice Tasks

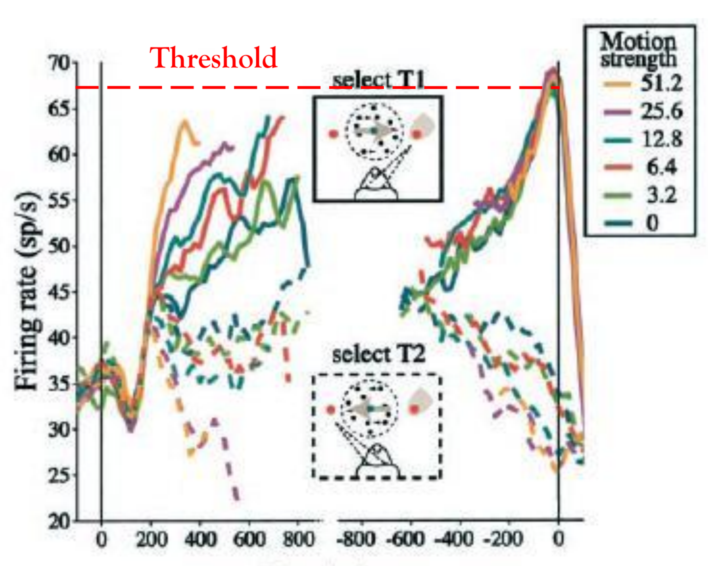


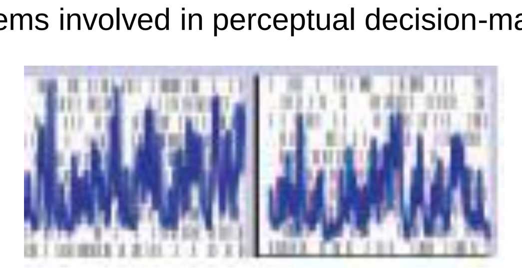
On each trial the primate is shown one of two stimuli, drawn at random. It must identify the direction (L or R) in which the majority of dots are moving. The experimenter can vary the coherence of movement (% moving L or R). Correct decisions are rewarded with drops of juice. Primates respond by making a visual saccade towards L or R. Goal is to maximize rewards. Can respond at leisure (Reaction Time task : A) or at a fixed time (Fixed Viewing Duration task: B)

Experimental Results



Electrophysiological recordings identify neural systems involved in perceptual decision-making

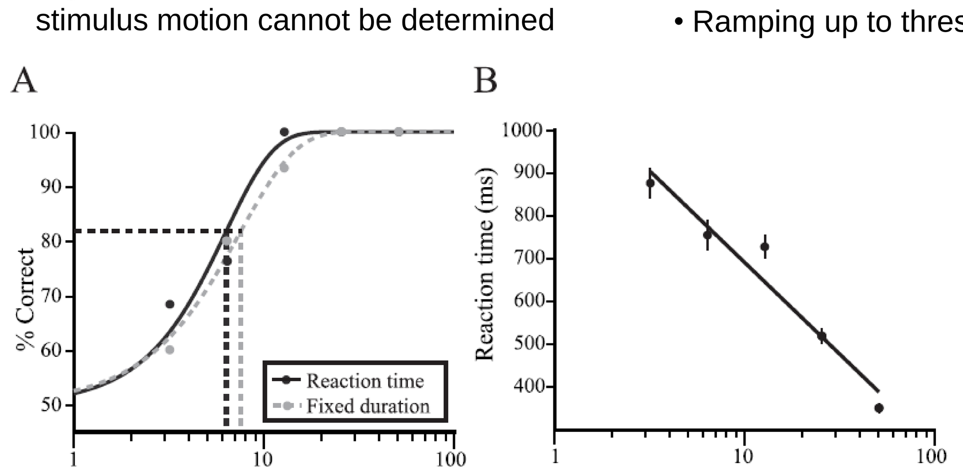


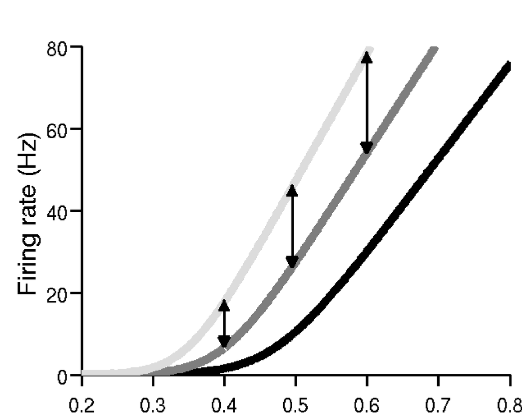


Signals in Area MT are noisy. Direction of stimulus motion cannot be determined

Recordings from the primate Lateral Intra-Parietal Cortex (LIP) demonstrate that neurons integrate evidence favoring each alternative until a threshold is crossed. The time at which it is crossed is correlated with the behavioral Reaction Time.

- Ramping up to threshold faster for higher Motion Strength



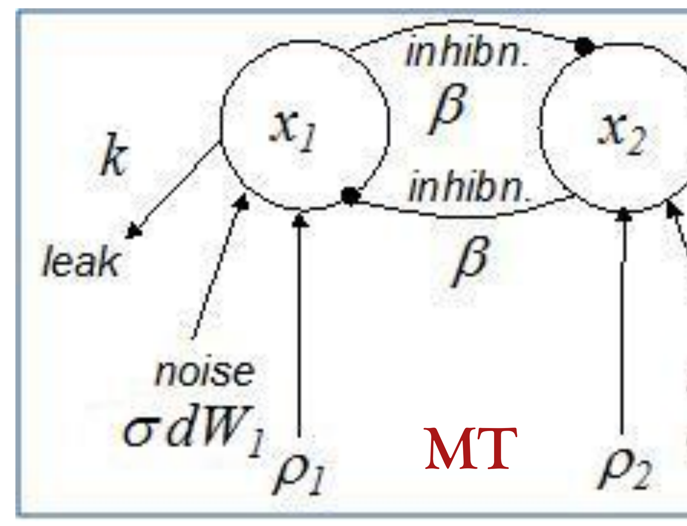


A) Psychometric function relating Accuracy to % coherence:
- Higher the coherence, greater the Accuracy

B) Faster Reaction Times (equivalent to faster ramping to threshold) for higher motion strength

Input Current – Output Firing Rate Transfer Function
- Changing the Gain affects the Slope

Leaky Competing Accumulator Model



2 Dimensional Model

2 neural populations selective towards L / R

- LIP neurons integrate sensory evidence
  - Decision units x_i
- Receive noisy signals from the MT
  - Input mean activities ρ_i
- Wiener Noise: adds independent random increments $dW_j = \sqrt{\frac{dt}{\tau}}N(0,1)$
- Leakage (k) in evidence accumulation
- Neural populations inhibit (β) each other

Stochastic Differential Equations

$$dx_1 = [\rho_1 - kx_1 - \beta f(x_2)]\frac{dt}{\tau} + \sigma dW_1$$
$$dx_2 = [\rho_2 - kx_2 - \beta f(x_1)]\frac{dt}{\tau} + \sigma dW_2$$
$$x_1(0) = x_2(0) = 0$$

Parameters

$\rho_1 + \rho_2 = 1 \quad \rho_1 - \rho_2 = C \in [-1, 1]$

C ≡ Coherence $\quad k, \beta > 0$

Input Current ⟶ Output Firing-Rate Functions

Most Non-Linear ⟶ **Linear**

Logistic
$$f(x) = \frac{1}{1 + exp(-4g(x-b))}$$

Upper Cut-Off
$$f(x) = \begin{cases} 0 & \text{if } x \in (-\infty, b - \frac{1}{2g}) \\ x & \text{if } x \in [b - \frac{1}{2g}, b + \frac{1}{2g}] \\ 1 & \text{if } x \in (b + \frac{1}{2g}, \infty) \end{cases}$$

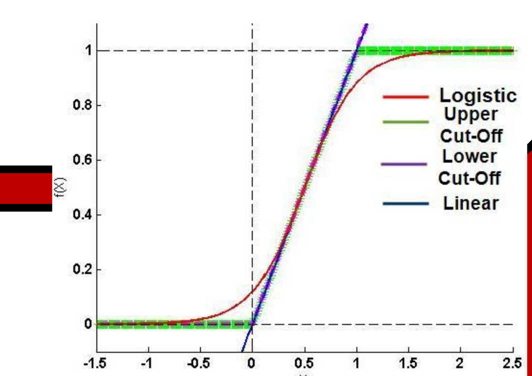
Lower Cut-Off
$$f(x) = \begin{cases} 0 & \text{if } x \in (-\infty, b - \frac{1}{2g}) \\ x & \text{if } x \in [b - \frac{1}{2g}, \infty) \end{cases}$$

Truncated Activation
$$f(x) = x \qquad x_i(t) = \max(x_i(t), 0)$$

Linear
$$f(x) = x$$

$f(b) = \frac{1}{2} \quad f_g'(b) = g$

Gain $g = 1$ Shift $b = 0.5$



Phase Plane Analysis

Consider Deterministic System

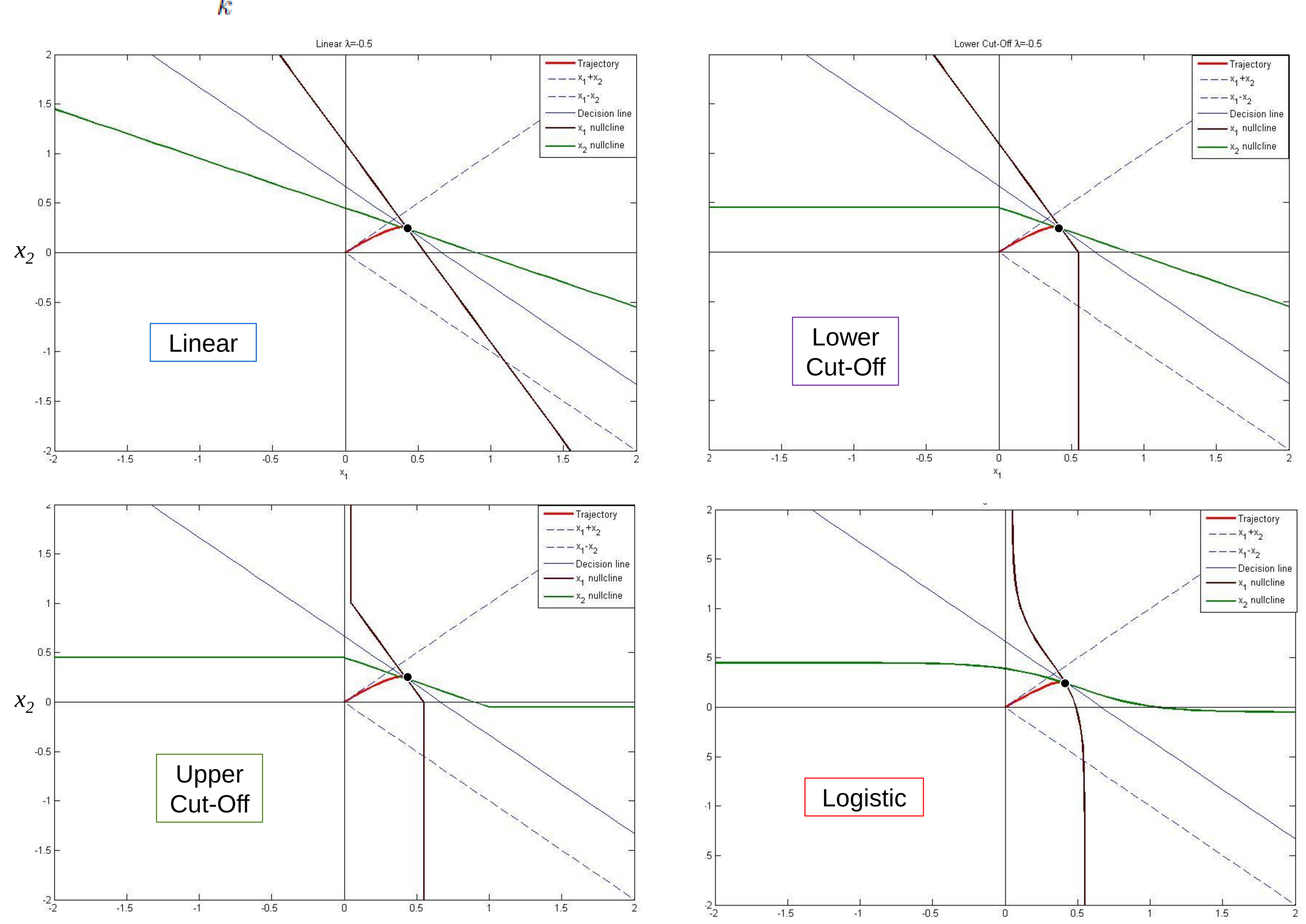
$$\frac{dx_1}{dt} = 0 \quad x_1 = \frac{\rho_1 - \beta f(x_2)}{k}$$

NULLCINES

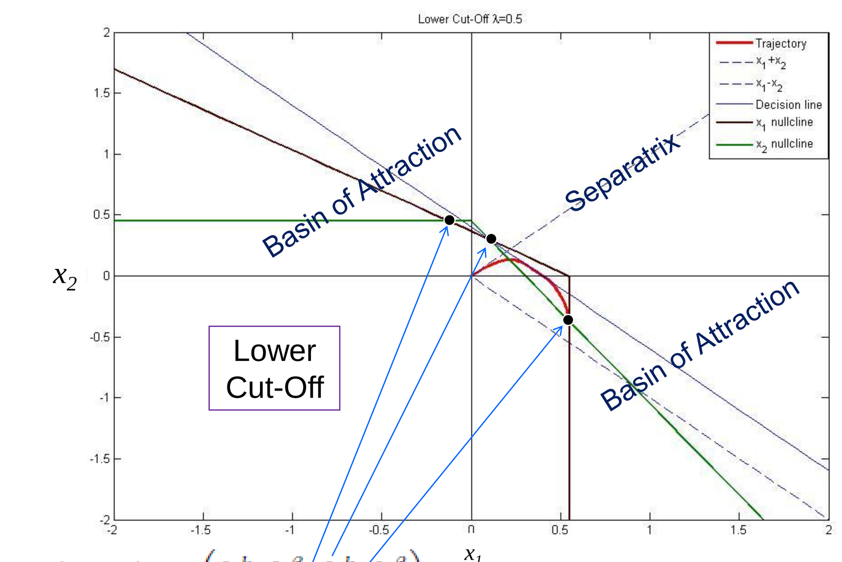
$$\frac{dx_2}{dt} = 0 \quad x_2 = \frac{\rho_2 - \beta f(x_1)}{k}$$

Intersection of Nullclines yields Fixed Point

- Fixed Point important in Fixed Viewing Duration task: Decision must be stored in working memory during delay period



Leak greater than Inhibition: $\beta < k$ Stable Unique Fixed Point, can be quadrant I, II or IV



$(x_1, x_2)^* = \left(\frac{\rho_1 k - \rho_2 \beta}{k^2 - \beta^2}, \frac{\rho_2 k - \rho_1 \beta}{k^2 - \beta^2}\right)$ Saddle Point $\quad C \in \left(-\frac{\beta - k}{\beta + k}, \frac{\beta - k}{\beta + k}\right)$

$(x_1, x_2)^* = \left(\frac{\rho_1 k - \rho_2 \beta}{k^2}, \frac{\rho_2}{k}\right)$ Sink

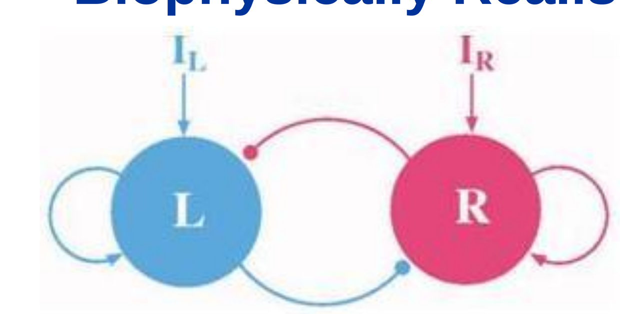
$(x_1, x_2)^* = \left(\frac{\rho_1}{k}, \frac{\rho_2 k - \rho_1 \beta}{k^2}\right)$ Sink

3 Fixed Points

LCA Type	Linear	Lower Cut-Off
$\beta < k$ Leaky	Unique Stable Fixed Point	Unique Stable Fixed Point
$\beta = k$ Balanced	No Fixed Point (C ≠ 0)	Unique Stable Fixed Point
$\beta > k$ Inhibitory	Unique Unstable Fixed Point	1, 2 or 3 Fixed Points Sinks or Saddles

Coherence *C* determines Quadrant / Stability

Biophysically Realistic Recurrent Network Model



Reduced Two-Variable Approximation of Spiking Neuronal Network Model

- Similar to LCA model, but biophysically realistic
- Recurrent Self-Excitation ($J_{LL} = J_{RR}$)
- Mutual Inhibition ($J_{LR} = J_{RL}$)
- Receives Input Current from MT due to Target and Motion Stimulus

$$r_i = f(I_i) = \frac{aI_i - b}{1 - \exp[-d(aI_i - b)]}$$

Input Current (*I*) Output Firing-Rate (*r*) Function

$$I_{L,tot} = J_{LL}S_L - J_{LR}S_R + I_{motion,L} + I_{target} + I_{noise,L}$$
$$I_{R,tot} = J_{RR}S_R - J_{RL}S_L + I_{motion,R} + I_{target} + I_{noise,R}$$

Total Input Current

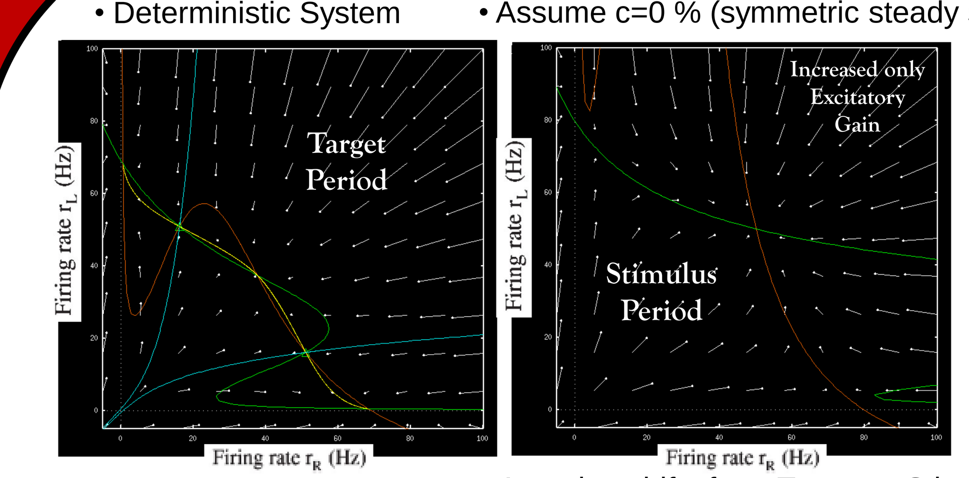
$$\frac{dS_i}{dt} = -\frac{S_i}{\tau_S} + (1 - S_i)\gamma f(I_i)$$

Dynamical Equation

$I_{motion,i} = J_{A}\mu_0\left(1 + \kappa\frac{c}{100}\right)$ *c* ≡ % Coherence

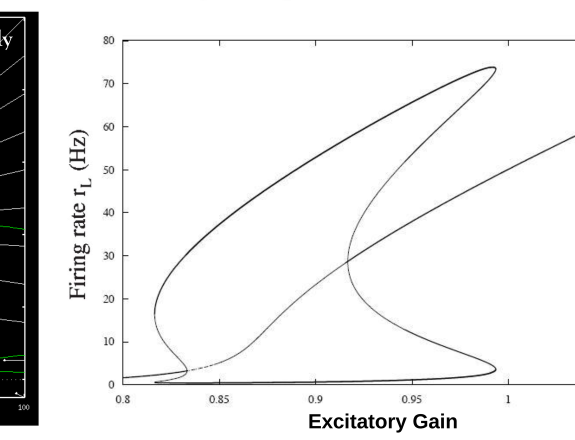
Phase Plane Analysis

- Deterministic System
- Assume c=0 % (symmetric steady state) $S_1^* = S_2^*$



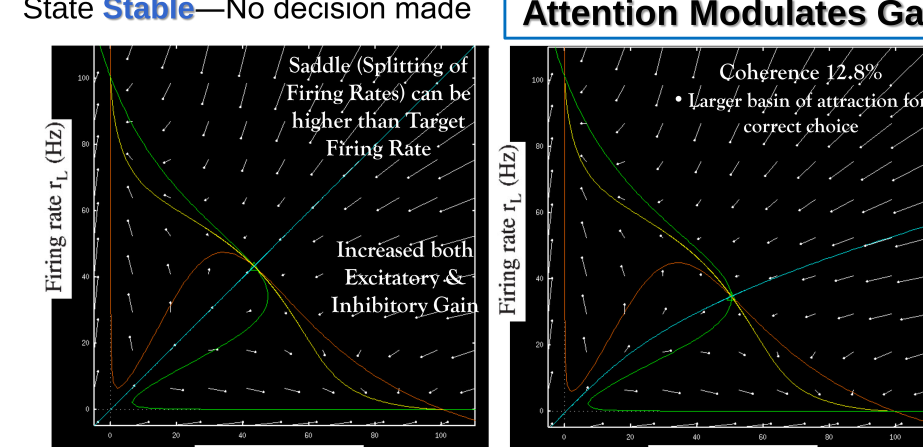
- Target period: Symmetric Steady State Stable—No decision made

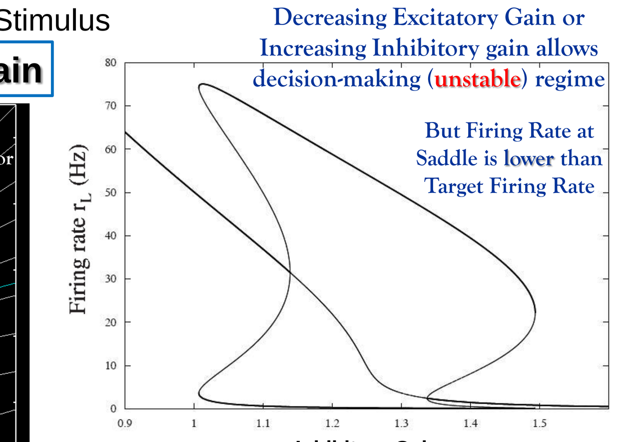
Attention shifts from Target to Stimulus



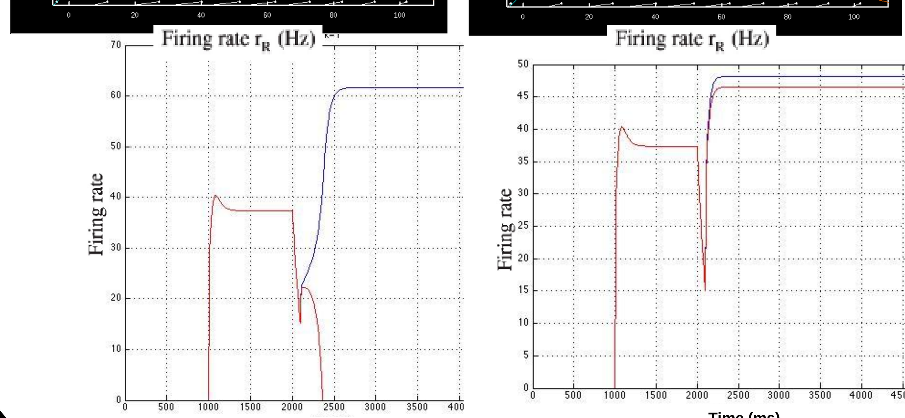
Attention Modulates Gain

Decreasing Excitatory Gain or Increasing Inhibitory gain allows decision-making (unstable) regime





$$\alpha_1 - \alpha_2 = k_w' - \frac{k' c_I}{1 + k_2 c_I}$$
$$J_{LL} = J_{RR} = \alpha_1 \quad J_{LR} = J_{RL} = \alpha_2$$



Stimulus Motion Period:
- Increase in Gain of both Excitatory and Inhibitory cells necessary for Symmetric steady state to be a **Saddle**: Categorical Decision-Making possible
- Time-Varying Gain Modulation Gates the Network such that the population selective for the correct direction wins

Increasing Excitatory Gain alone prevents Winner-Take-All Decision-Making

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